Refraction Statics

- Plus-Minus method;
- Generalized Linear Inverse (GLI, tomographic) method;
- Delay-time ('*time-term*') method;
- L_1 and L_2 norms.

- Reading:
  - Yilmaz (refraction statics section and Appendix C in the new edition)

Delay time

- Consider a nearly horizontal, shallow interface with strong velocity contrast (a typical case for weathering layer).
  - In this case, we can separate the times associated with the source and receiver vicinities: $t_{SR} = t_{SX} + t_{XR}$.
- Relate the time t_{SX} to a time along the refractor, t_{BX}:

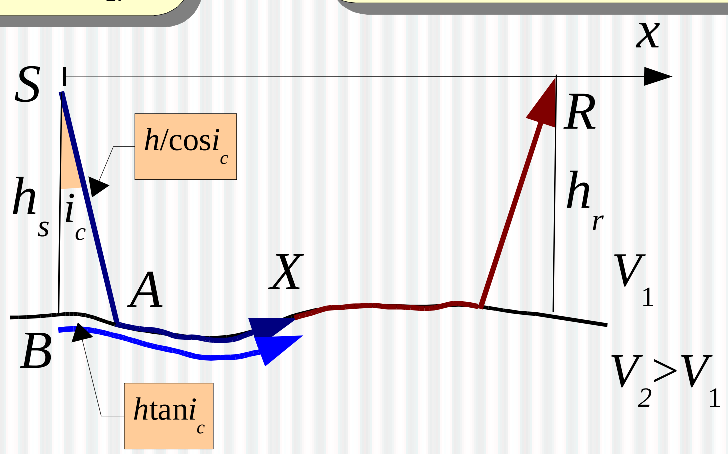
$$t_{SX} = t_{SA} - t_{BA} + t_{BX} = t_{S\ Delay} + x/V_2.$$

$$t_{S\ Delay} = \frac{SA}{V_1} - \frac{BA}{V_2} = \frac{h_s}{V_1 \cos i_c} - \frac{h_s \tan i_c}{V_2} = \frac{h_s}{V_1 \cos i_c}(1 - \sin^2 i_c) = \frac{h_s cosi_c}{V_{1.}}$$

Note that $V_2 = V_1/\sin i_c$

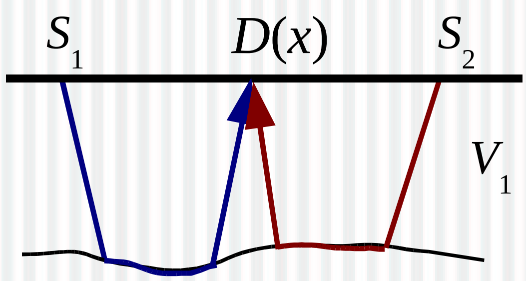
- Thus, source and receiver delay times are:

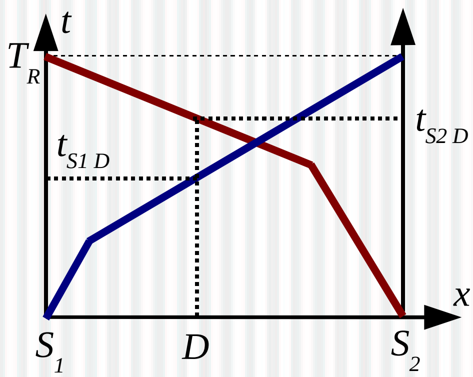
$$t_{S,R\ Delay} = \frac{h_{s,r} cosi_c}{V_{1.}}$$ and $$t_{SR} = t_{S\ Delay} + t_{R\ Delay} + \frac{SR}{V_{2.}}$$



Plus-Minus Method
(Weathering correction; Hagedoorn)

- Assume that we have recorded two head-waves in opposite directions, and have estimated the velocity of overburden, $V_{1.}$
  - How can we map the refracting interface?





- Solution:
  - Profile $S_1 \rightarrow S_2$: $t_{S_1 D} = \frac{x}{V_2} + t_{S_1} + t_D;$
  - Profile $S_2 \rightarrow S_1$: $t_{S_2 D} = \frac{(S_1 S_2 - x)}{V_2} + t_{S_2} + t_{D.}$
  - Form PLUS travel-time:

$$t_{PLUS} = t_{S_1 D} + t_{S_2 D} = \frac{S_1 S_2}{V_2} + t_{S_1} + t_{S_2} + 2t_D = t_{S_1 S_2} + 2t_{D.}$$

Hence: $t_D = \frac{1}{2}(t_{PLUS} - t_{S_1 S_2}).$

  - To determine i_c (and depth), still need to find V_2.

Plus-Minus Method (Continued)

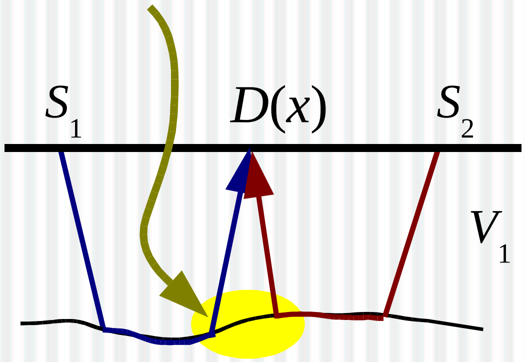
- To determine V_2:
  - Form MINUS travel-time:

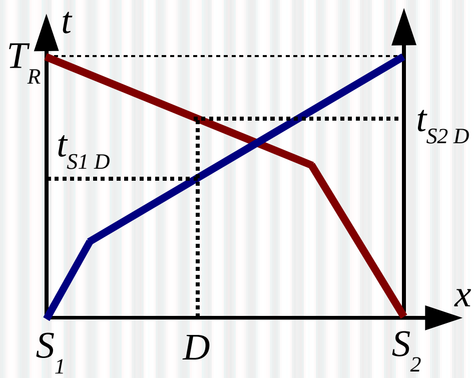
this is a constant!

$$t_{MINUS} = t_{S_1 D} - t_{S_2 D} = \frac{2x}{V_2} - \frac{S_1 S_2}{V_2} + t_{s_1} - t_{s_2}.$$

Hence: $$slope[t_{MINUS}(x)] = \frac{2}{V_2}.$$

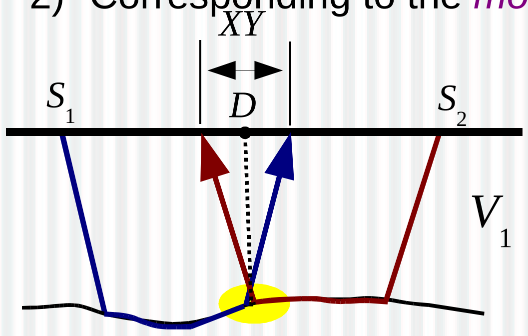
  - The slope is usually estimated by using the *Least Squares method*.
- Drawback of this method – averaging over the pre-critical region.

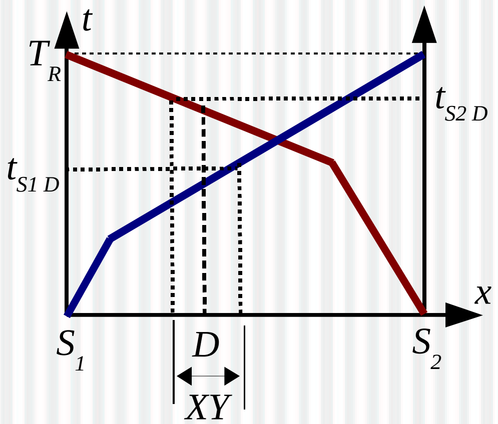




Generalized Reciprocal Method (GRM)

- Introduces offsets ('*XY*') in travel-time readings in the forward and reverse shots;
  - so that the imaging is targeted on a compact interface region.
- Proceeds as the plus-minus method;
- Determines the '*optimal*' *XY:*
  1) Corresponding to the most linear *velocity analysis function;*
  2) Corresponding to the *most detail* of the refractor.





- The *velocity analysis function*:

$$t_V = \frac{1}{2}\left(t_{S_1 D} - t_{S_2 D} + t_{S_1 S_2}\right),$$

should be linear, slope $= 1/V_2$;

- The *time-depth function*:

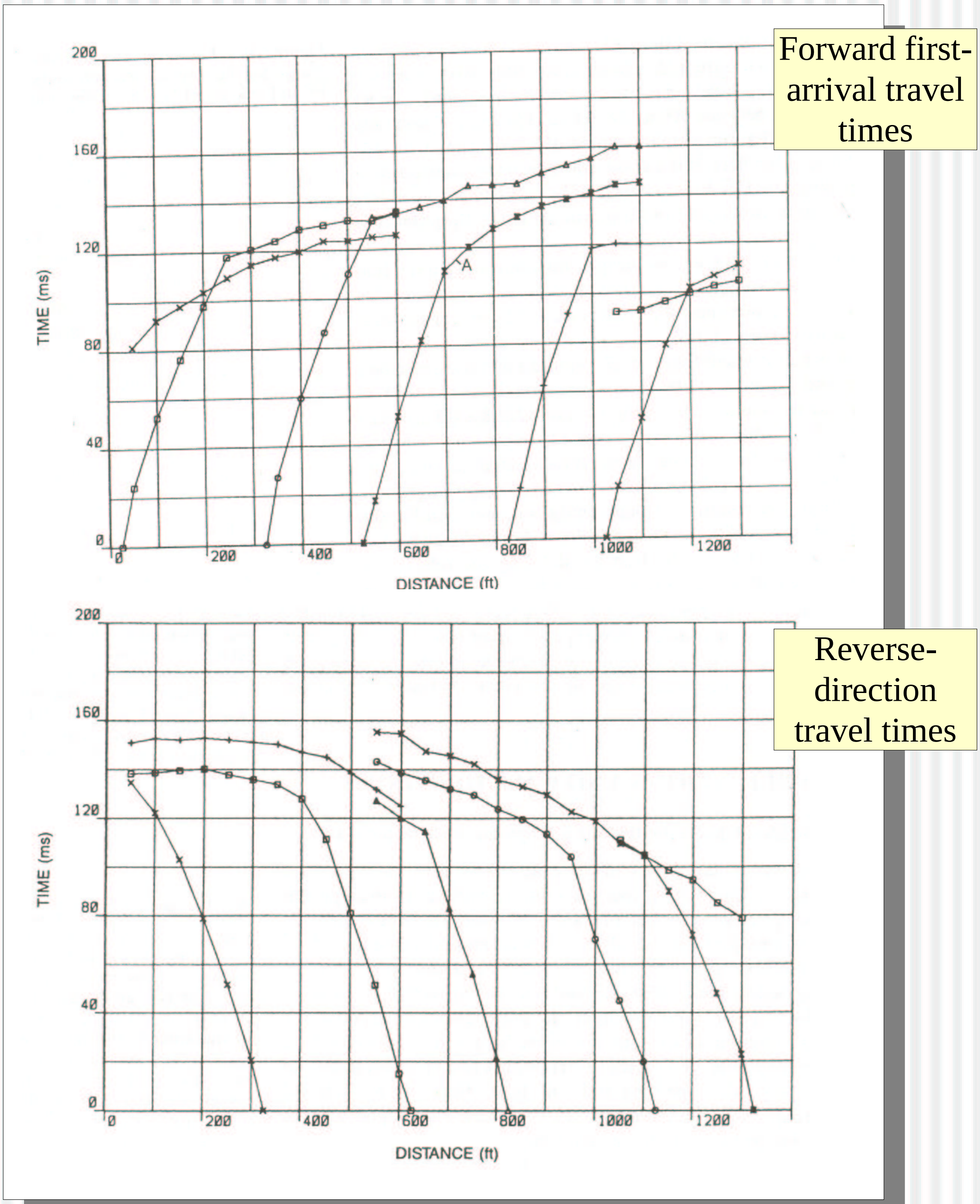
$$t_D = \frac{1}{2}\left(t_{S_1 D} + t_{S_2 D} - t_{S_1 S_2} - \frac{XY}{V_2}\right).$$

this is related to the desired image:

$$h_D = \frac{t_D V_1 V_2}{\sqrt{V_2^2 - V_1^2}}$$

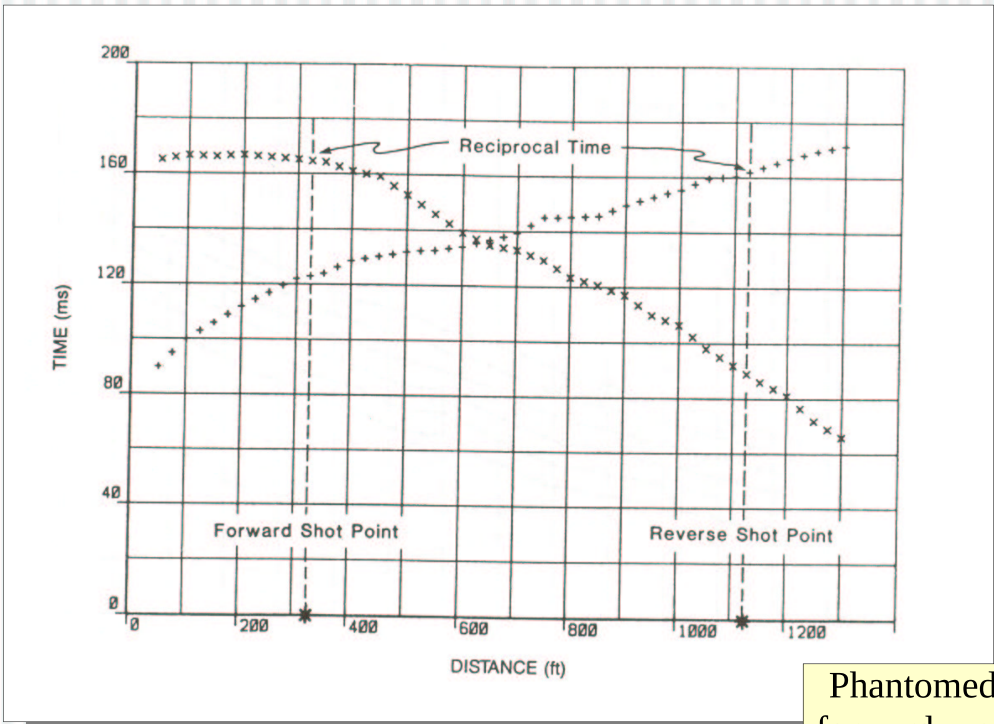
Example:
Refraction survey over a proposed waste disposal site

(Lankston, 1990), recorded travel times



Refraction survey over a proposed waste disposal site

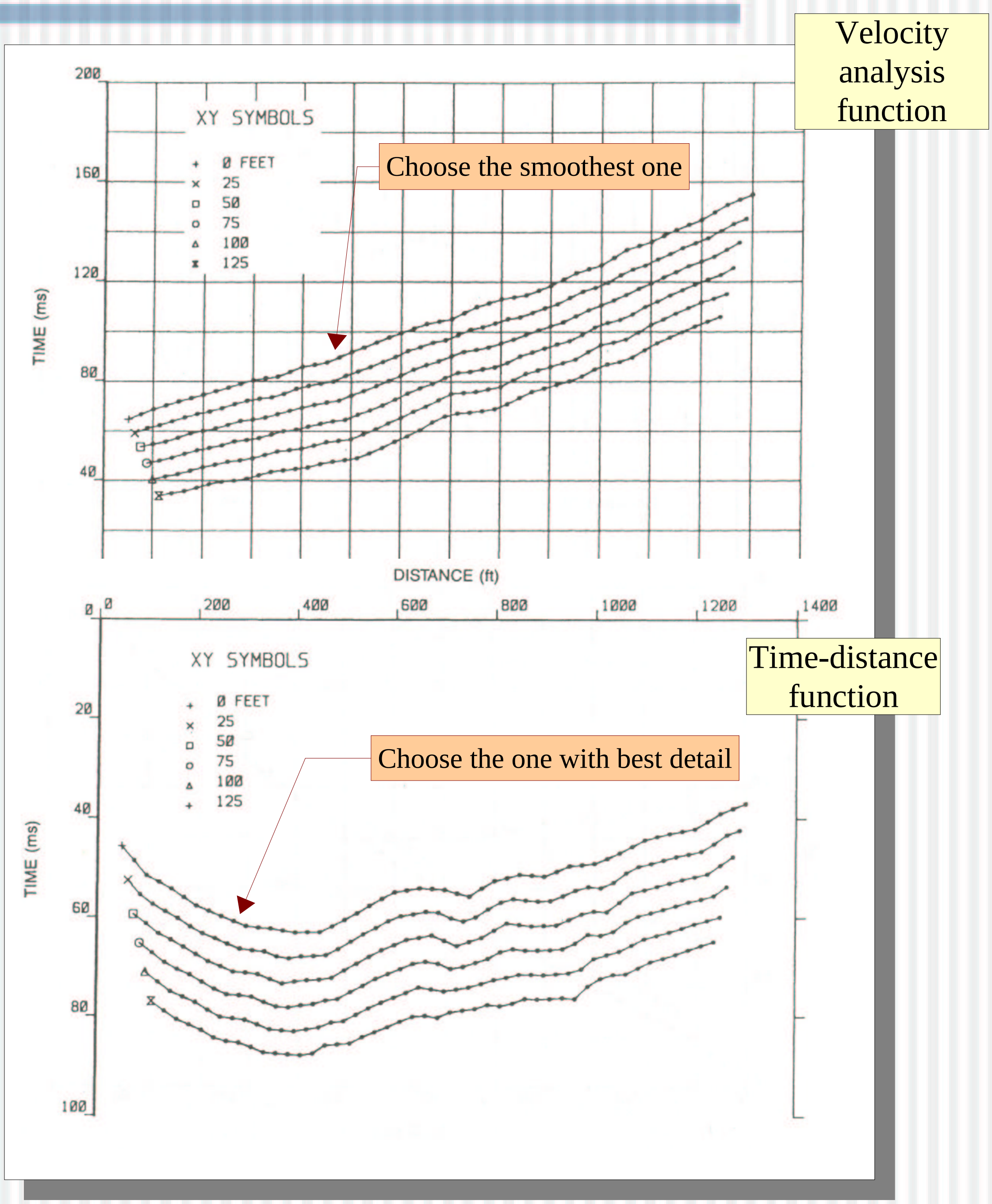
(Lankston, 1990)



Phantomed forward- and reverse-direction first-arrival travel times

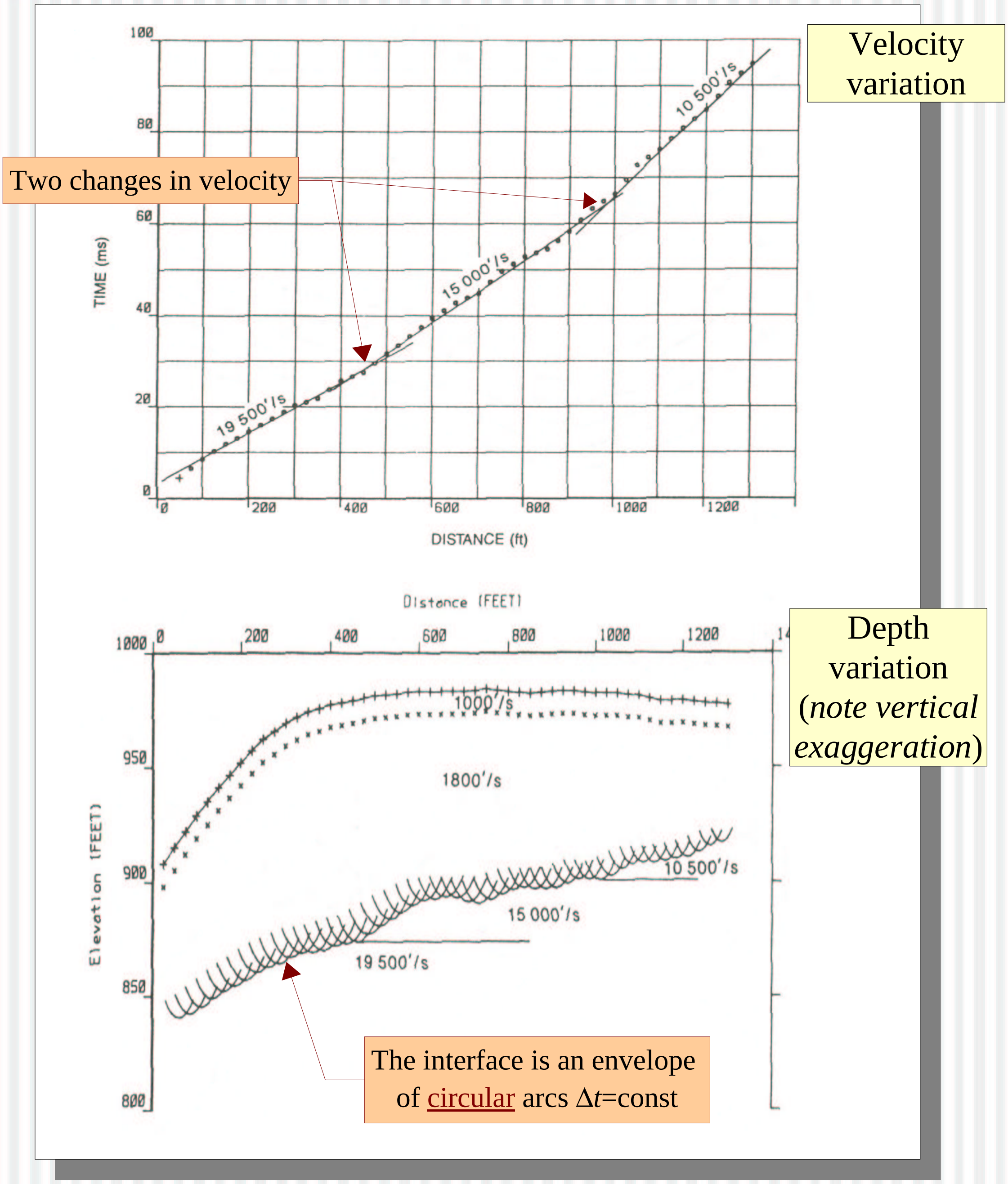
Refraction survey over a proposed waste disposal site

(Lankston, 1990), velocity analysis



Refraction survey over a proposed waste disposal site

(Lankston, 1990), results



Generalized Linear Inverse

- Usually *overdetermined* travel-time inversion problem
  - Impossible to fit all times at once;
  - Trying to minimize $(t_{ij} - t_{ij}^{observed})^2$
- Solved using *damped least squares* method.
- In matrix form:

$$\boldsymbol{A}\,\boldsymbol{m} = \delta\,\boldsymbol{t},$$

$$(\boldsymbol{A}^T\boldsymbol{A})\,\boldsymbol{m} = \boldsymbol{A}^T\,\delta\,\boldsymbol{t},$$

$$\boldsymbol{m} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\,\delta\,\boldsymbol{t}.$$

- Regularized using *damping parameter*, μ:

$$\boldsymbol{m} = (\boldsymbol{A}^T\boldsymbol{A} + \mu\,\boldsymbol{I})^{-1}\boldsymbol{A}^T\,\delta\,\boldsymbol{t}.$$

Generalized Linear Inverse Method

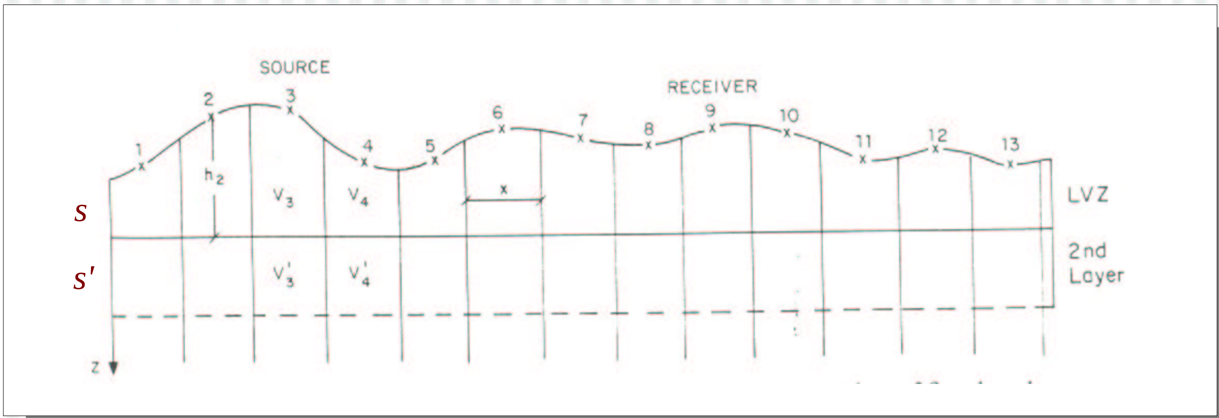
(Hampson and Russel (1984), Amorim et al. (1987))

- Tomographic model
- Parameterized in terms of slownesses within the weathering (LVZ) layer

$$t_{ij} = s_i \frac{H_i}{\cos\theta_i} + s_j \frac{H_j}{\cos\theta_j} + s'_i\left(\frac{x}{2} - H_i \tan\theta_i\right) + s'_j\left(\frac{x}{2} - H_j \tan\theta_j\right) + \sum_{n=i+1}^{j-1} s'_n x$$

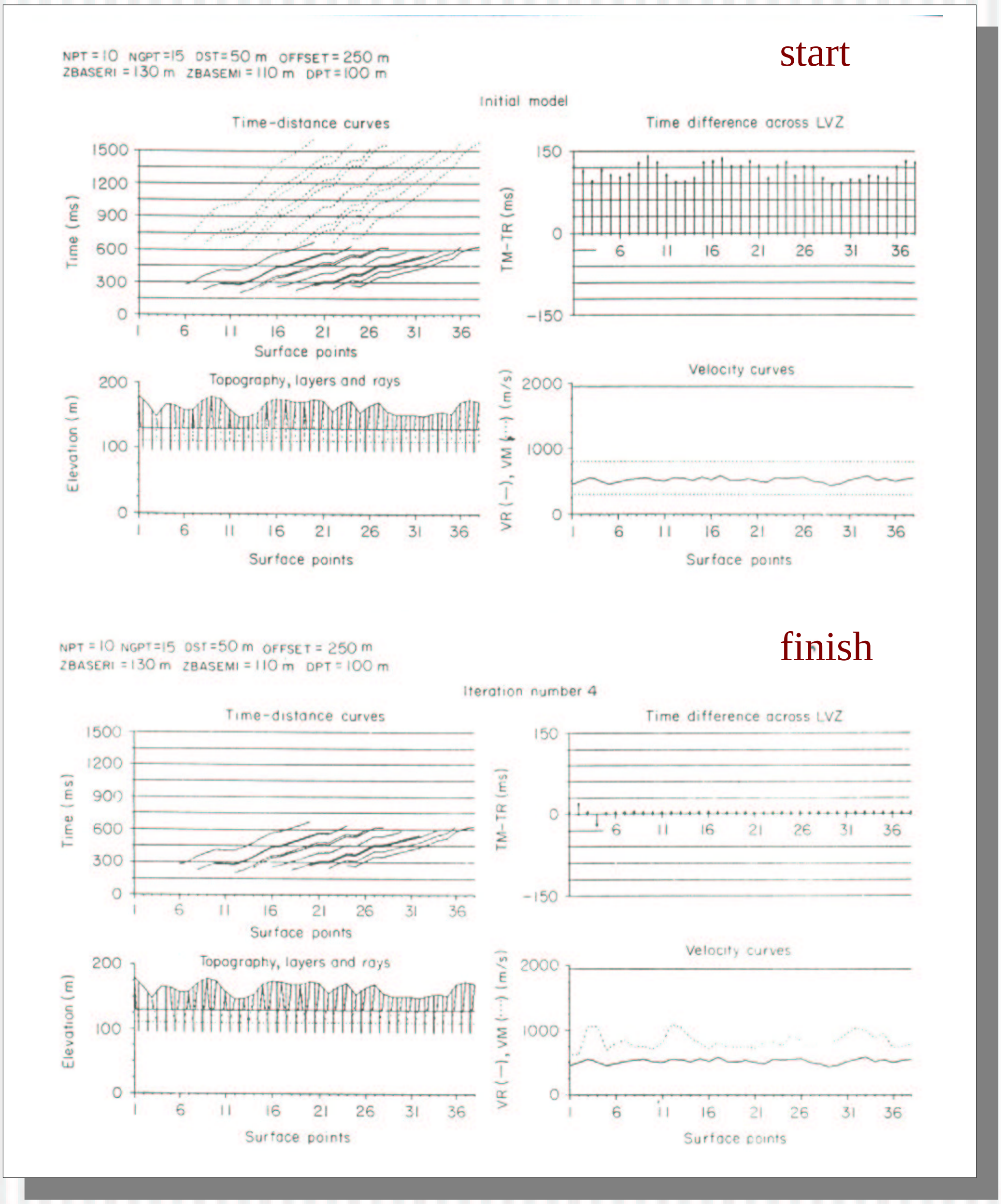
$$\theta = \arcsin\left(\frac{s'}{s}\right) = \arcsin\left(\frac{v}{v'}\right).$$

- These equations can be viewed in two ways:
  - fixed *H*, linear equations in *s* and *s'* (fixed-depth scheme);
  - fixed *s*, linear equations in *H* and *s'* (variable- depth scheme);



GLI Refraction Statics

- Iterations to reduce δt:



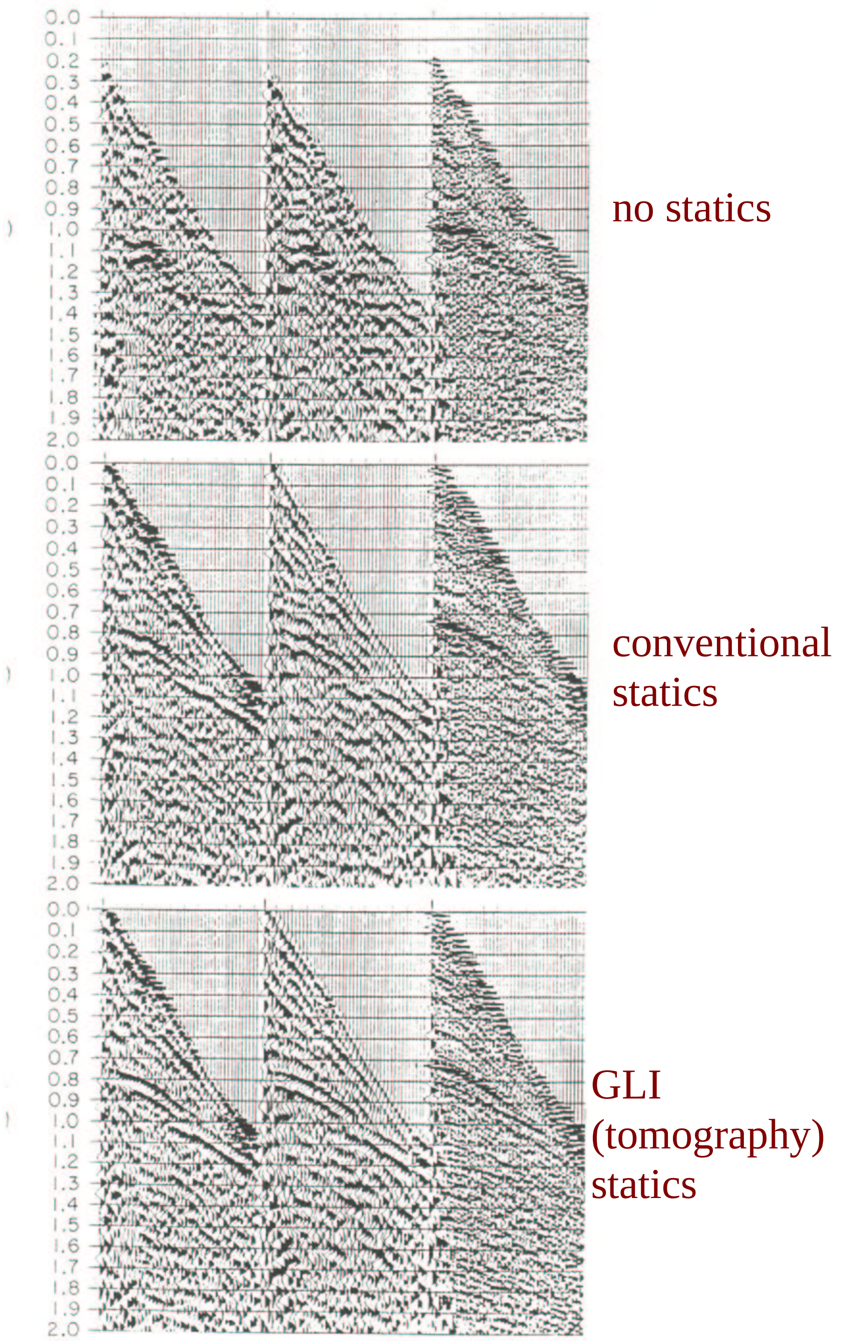
Delay-time (time-term) method

- Linear inverse model for delay times (Δt_S and Δt_R) and retractor slowness $s = 1/V$:

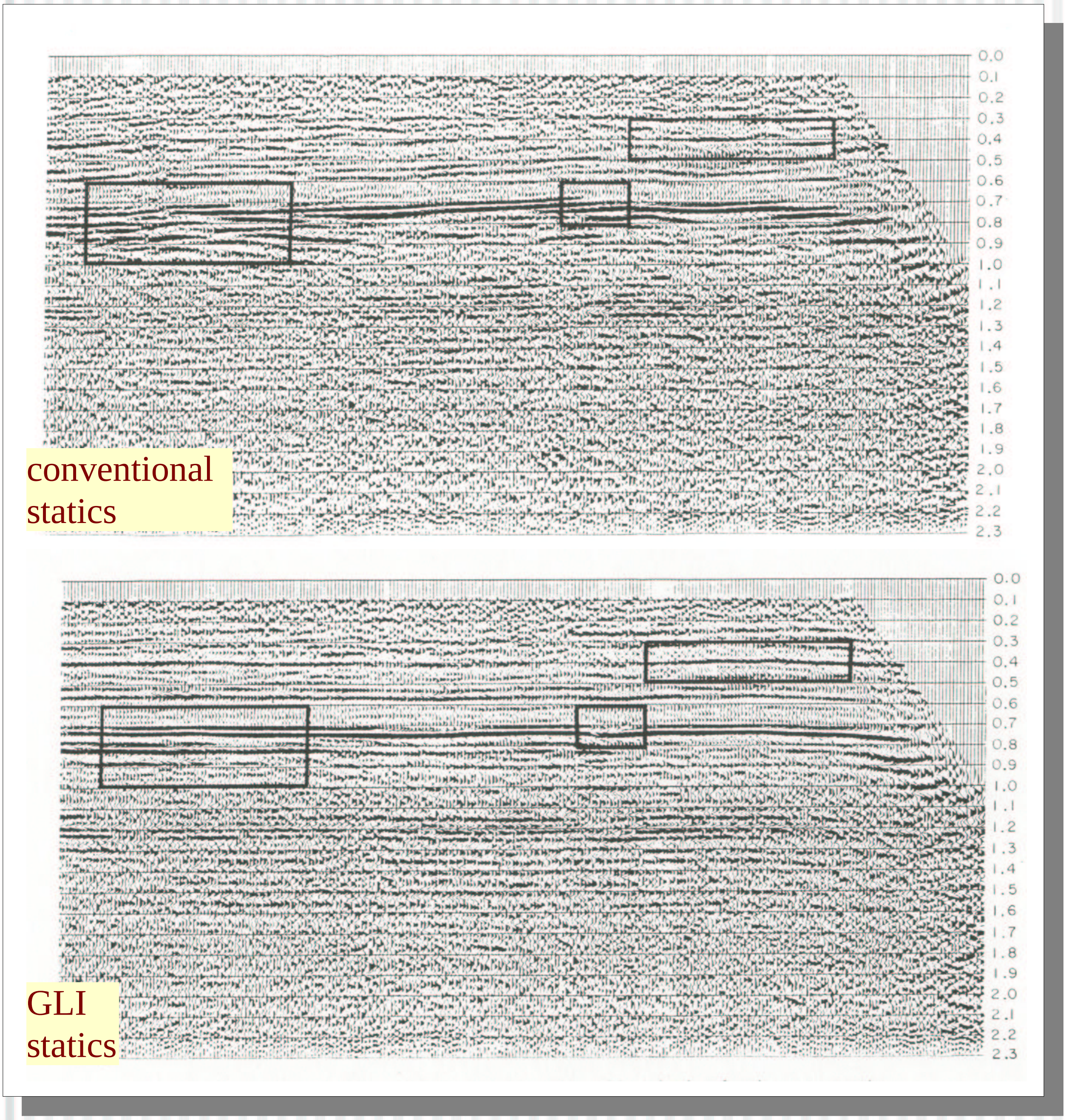
$$t_{SR} = \delta t_S + \delta t_R + \frac{x}{V} = \delta t_S + \delta t_R + sx.$$

- Δt_S and Δt_R are usually *surface consistent* (dependent on common locations only), and s can be *spatially variant*
  - Spatial variability described using a grid, similarly to GLI method.

Effects of statics in shot gathers



Effects of statics in stacked image



L_1 vs. L_2 Norms

- Most tomographic methods use the so-called L_2 norm: $(t_{ij} - t_{ij}^{observed})^2$
- However, L_2 is not the best norm:
  - sensitive to *outliers* (wild mis-picks).
- Other used norms are:
  - $L_1 = |t_{ij} - t_{ij}^{observed}|$ (more stable in respect to outliers);
  - $L_\infty = \max|t_{ij} - t_{ij}^{observed}|$ (extremely sensitive to outliers, the most stringent norm).
- L_1-norm is inverted using *linear programming* (simplex) techniques.
- For good picks (not plagued by outliers) the results of L_2 and L_1-norm inversion are very close.