MINIMUM MODEL, WEIGHTING AND M-L

In the last assignment you wrote a program to solve the basic inversion problem of fitting a line to data. In this assignment you will modify the program to allow for weighting of the model parameters and the data and then to generate a Marquardt-Levenberg solution.

First of all, modify your program so that it can do the minimum model solution. You will probably want a flag or switch to instruct the program to do minimum error or the minimum model. The minimum model solution is

$$m^{est} = K^T(KK^T)^{-1}d$$

Note that the minimum-length solution is for underdetermined models. For your overdetermined model in Assignment 1, the matrix inverse here will fail (rank of KKT equals only 2 with dimensions equal 30). Therefore, you need to use the damped minimum-length solution to obtain this m^est. See class notes.

In the minimum error program from assignment one, write in the code that allows for weighting the data. Remember that with no weighting - the solution you had last week - the weighting matrix would be the identity matrix, so your new program will also give the previous solution if you make the weighting matrix the identity matrix. The weighted data Gauss-Newton solution is

$$m^{est} = (K^T W_e K)^{-1} K^T W_e d$$

where W_e is the data weighting matrix.

Test this solution by giving the outliers in the assignment 1 data a very high weight and a very low weight compared to all the data, and observe how this affects the solution. For example, if one of the outliers was the tenth data point, then the weighting matrix could have 1's on the main diagonal, except for the tenth element, which would have a numerical value much higher than 1 or much less than 1. All the other elements would be zero.

Add more code so that your solution can now be Marquardt Levenberg. Don't forget to write it so that you can still do the classical Gauss Newton solution if you set $\alpha = 0$.

Again note that I call this solution "the least squares", and the "Gauss-Newton" name is reserved for solving nonlinear problems.

Here is the M-L solution

$$m^{est} = (K^T K + \alpha I)^{-1} K^T d$$

and of course if $\alpha = 0$ this reduces the Gauss-Newton solution.