Physical character of seismic viscoelasticity

Igor B. Morozov

Department of Geological Sciences, University of Saskatchewan, Saskatoon, SK S7N 5E2 Canada

Abstract

Despite its broad use in seismology and geodesy, the physical significance of the "viscoelastic" model of Earth's materials is still not well understood. Viscoelasticity is based on the concept of material memory, which implies either a departure from the classical physical framework of instantaneous interactions or presence of a rather specific type of internal variables within the medium. Nevertheless, such modifications are not needed, and the traditional mechanics successfully explains all "viscoelastic" effects in solids and fluids, including creep, wave attenuation, velocity dispersion, as well as many other phenomena. Viscoelasticity represents a phenomenological, quasi-static approximation to the full physical picture, which is only applicable to near-uniform media. Although this approximation is arguably sufficient in empirical materials science, it appears inadequate for seismic waves or oscillations of heterogeneous planetary bodies. "Memory" is not an inherent property of materials but only a time record of deformation for a given body. Contrary to what is often thought, the observed variations of material properties, such as the transitions from "unrelaxed" to "relaxed" moduli, are governed not by the time but by the thermodynamic conditions of deformation. Several important types of mechanical-energy dissipation cannot be accounted for by the material memory at all, for example: 1) wet porous rock can dissipate energy without macroscopic deformation, 2) fluid zones dissipate shear energy despite their shear moduli being equal zero, and 3) perfectly elastic rock also dissipates mechanical energy in spatially-heterogeneous environments. The quality factor (Q) is also hardly valid physically when viewed as a local property of the medium and related to viscoelastic moduli. Given the above ambiguities and limitations of the viscoelastic model, we recommend that rigorous physical approaches are used more often. These approaches explain the Earth's anelasticity by well-defined physical factors, such as the solid and pore-fluid viscosity, thermoelastic and kinetic effects, and also the effects of structure and scattering. These factors operate at all time scales and conditions, which allows developing a unified and quantitative picture of seismic and geodetic anelasticity.

Subject classification

04.06.09. Seismology: Waves and wave analysis

04.06.06. Seismology: Surveys, measurements, and monitoring

04.06.99. Seismology: General or miscellaneous

05.05.99. Mathematical geophysics: General or miscellaneous

1. Introduction

Among the various approaches to anelasticity in solids, the "viscoelastic" model, originally proposed by Bland [1960] and introduced in seismology by Anderson and Archambeau [1964] is broadly believed to be most relevant for Earth materials [*e.g.*, Romanowicz and Mitchell, 2009; Cormier, 2011]. This model is presented in all standard texts in seismology [*e.g.*, Dahlen and Tromp 1998, and Aki and Richards 2002] and Earth materials science [*e.g.*, Karato 2008]. Its central point consists in characterizing the anelasticity of crustal and mantle rocks by the concepts of material memory and quality factor (Q). The Q-based paradigm is now deeply imbued in minds of most geophysicists and used in practically all interpretations, forward modeling, inversion methods, and attenuation models [for example, the global Preliminary Reference Model (PREM) by Dziewonski and Anderson 1981]. This paradigm also determines the correlations of seismic anelasticity with mantle viscosities derived from geodynamic studies [Anderson 1965; Karato and Wu 1993; Sato 1991]. Under the influence of seismology, models of tidal anelasticity are also formulated in terms of the viscoelastic Q [*e.g.*, Agnew 2009].

Nevertheless, despite such broad acceptance, it is also useful to see that the viscoelastic model in geophysics still remains a mixture of intuitive analogies and axiomatic (and somewhat risky) mathematical constructions. The physical basis of its application to the Earth appears to be not well studied and understood. In particular, it is generally unnoticed that generalizations of rock-creep experiments in the lab, from which this model was inferred, cannot be automatically applied to seismic waves and free oscillations of the Earth. In the present paper, we analyze the limitations of seismic viscoelasticity and attempt identifying its place with respect to the traditional solid- and fluid-state physics. Unfortunately, this leads us to questioning the validity of the core concepts of this model, which are the material memory and Q.

Succinctly, the theory of viscoelasticity can be described as follows. Most materials are assumed to possess a "fading memory," which means that responses of stresses to deformations (and *vice versa*) are localized in space but delayed in time. This memory is expected to span a time range as broad as 10^{-9}–10^{-6} s to 10^{13} s [Jackson *et al*. 2005]. In the frequency domain, this property is described by extending the elastic modulus M of the material into the complex plane and making it frequency-dependent. Such an extension is achieved by noting that for a harmonic oscillation at frequency ω, both the elastic (σ_e) and viscous stresses (σ_f) can be formally rendered in the form of the elastic Hooke's law:

$$\sigma_e(\omega) = m\varepsilon(\omega), \tag{1}$$

$$\sigma_f(\omega) = \eta\dot{\varepsilon}(\omega) = -i\omega\eta\varepsilon(\omega), \tag{2}$$

where ε is the strain, m is the elastic modulus, η is the viscosity, and overdot indicates the time derivative. By combining the two stresses: $\sigma = \sigma_e + \sigma_f$, and constructing a complex "viscoelastic" modulus $M(\omega) = m - i\omega\eta$, an equation similar to (1) is obtained, which incorporates the internal friction via the imaginary and frequency-dependent part of M:

$$\sigma(\omega) = M(\omega)\varepsilon(\omega), \tag{3}$$

This extension of the elastic-medium constitutive equation (1) to the anelastic case is called the correspondence principle [Bland 1960]. At this point, the model is further generalized from the original case of $m = \text{const}$ and $\eta = \text{const}$ above to an arbitrary $M(\omega)$ for $\omega > 0$. For $\omega \leq 0$, the values of M are constrained by ε and σ being real in the time domain, which requires that $M(\omega) = \bar{M}(-\omega)$, where the overbar denotes the complex conjugation. The tangent of the complex argument of M is denoted as the inverse of the "quality factor:"

$$Q^{-1}(\omega) \equiv -\frac{\operatorname{Im} M(\omega)}{\operatorname{Re} M(\omega)}, \tag{4}$$

and accordingly:

$$M(\omega) = \operatorname{Re} M(\omega)\left[1 - iQ^{-1}(\omega)\right]. \tag{5}$$

Thus, the two fundamental properties describing a viscoelastic solid are $m = \operatorname{Re}M$, which is the frequency-dependent elastic modulus, and Q^{-1} describing the dissipation. Both of these quantities are taken at $\omega \geq 0$. Note that from the original expressions (1) and (2), the Q^{-1} is a proxy for the ratio of the viscosity to the elastic modulus, and it is proportional to the frequency:

$$Q^{-1}(\omega) = \frac{\eta}{m}\omega. \tag{6}$$

In the time domain, ReM and Q^{-1} correspond to a time-dependent "fading memory" of the material, $M(t)$. Finally, in order to ensure causality ($M(t) \equiv 0$ for $t \leq 0$), Re$M(\omega)$ and Im$M(\omega)$ must satisfy the Kramers-Krönig integral identities [Dahlen and Tromp 1998].

The viscoelastic model above is constructed by generalizing the constitutive equation (1) into (3), and it is rarely noticed that this procedure does not completely agree with physics. It is often unclear whether this model: (i) denies the classical mechanical framework, (ii) extends it, or (iii) represents some type of approximation to it. Regarding the first of these possibilities, the need for introducing a "fading memory" $M(t)$ implies that the mechanical framework of instantaneous interactions is regarded as inadequate [Fabrizio and Morro 1992]. As shown in section 4.1 below, if memory $M(t)$ is implemented mechanically, it is achieved at the cost of disregarding spatial flows and wave processes within the medium. At the same time, there exists no real reason for such denial of the mechanical framework for Earth materials. The experimental base of viscoelasticity principally consists of rock-creep measurements in the lab, and such observations can be readily explained by viscosity or thermoelastic relaxation and without invoking a fading memory. Models of poroelastic media [*e.g.,* Müller *et al*. 2010] also show that mechanical systems exhibit all of the observed "viscoelastic" properties, such as creep, wave velocity dispersion, or the frequency-dependent apparent Q. Therefore, it is clear that rock creep and seismic waves *can* be described by the usual "instantaneous" mechanical interactions. This is also done with much better precision and revealing many specific details of the dissipation mechanisms instead of the "blanket" $Q(\omega)$ or $M(t)$.

With regard to the possibility (ii) above, the mathematical conjecture transforming eq. (1) into (3) is not in the same class with the fundamental principles of mechanics, such as the energy, Hamiltonian action, and the laws of thermodynamics. The physical framework for solids and fluids is well known [*e.g.*, Landau and Lifshitz 1976, 1986, 1987] and used in the more general treatments of materials [Hayden *et al*. 1965]. This approach explains the full range of physical phenomena from mechanics of particles to statistical physics, thermodynamics, and physical kinetics [Lifshitz and Pitaevskii 1981]. An application of this theory to deep-Earth and planetary conditions are given in Morozov [submitted]. This framework does not require extensions by the correspondence principle; on the contrary, the notion of the complex-valued modulus (3) goes sharply against the first principles of continuum mechanics. Moreover, the correspondence principle itself fails when applied to heterogeneous anelastic media [Morozov 2011a].

Therefore, if viewed as a valid theory of anelastic or plastic solids, viscoelasticity can only represent option (iii) above, *i.e.* be some type of approximation to the full physical description. The character of this approximation may not be easy to determine, which is one of the goals of the present paper.

The focus of our analysis is on the consistency and adequacy of the theoretical concept and not on data fitting. Unfortunately, the data available from seismic attenuation studies are limited and can often be modeled in various ways, including purely empirical time dependences of deformations and moduli [*e.g.,* Andrade 1910]. Viscoelsticity can be described as a semi-empirical approach, in which a specialized quantity (Q^{-1}) is used to describe the energy-dissipation data, and a similar empirical fitting is achieved in the frequency domain [Liu *et al*. 1976]. This creates a richly (likely over-) parameterized model which is usually capable of explaining the available attenuation data well. However, the interpretations of the *in situ* anelastic properties of materials in terms of material memory and *Q* may still be poorly constrained and justified.

The material *Q* in eq. (4) (together with derived attributes $q = Q^{-1}$ and $t^* = \int Q^{-1} dt$) is the commonly used but conceptually challenging constitutive parameter intended to characterize the anelasticity of materials. Usually, it is explained from an intuitive analogy with mechanics or acoustics, and yet the properties of energy dissipation in mechanics are not always well understood. Therefore, in section 2, we review the meanings of *Q* mechanics and try explaining the analogies on which the definition of the seismic material *Q* (4) is based. As shown there, the principal extensions from the mechanical to seismic *Q* consist in using non-resonant oscillations and arbitrary frequencies. Such extensions can be carried out in several ways, which causes significant ambiguity in the frequency dependencies of *Q*. We also show that it is important to differentiate between the apparent (observed) and the *in situ Q*, and that unlike its mechanical counterparts, parameter *Q should not* be viewed as a local property of the material.

In section 3, we discuss some limitations of the viscoelastic model for realistic solids. As illustrated on two simple thought experiments, the viscoelastic model (3)–(4) *does not* account for several most common physical mechanisms of anelasticity. In section 4, we compare this model to the physical descriptions of solid and fluid media. As shown there, the viscoelastic model represents an effectively quasi-static approximation to the rigorous model, applicable to nearly-uniform media. Arguably, this approximation appears suitable for describing rock creep experiments in the lab or for developing kinetic models [Nowick and Berry 1972; Cooper 2002]. In such cases, only responses to spatially uniform stresses are considered, and only a single *Q* value is needed for both the apparent and *in situ* properties. However, in heterogeneous media and/or deformation fields, such as encountered in seismic or tidal problems, this model presents the spatial flows and wave processes within the medium as “material memory” occurring at each of its points. Unfortunately, this picture is physically inadequate and may be grossly misleading, even if viewed as pure phenomenology.

2. The quality factor

Compared to the *in situ*, material Q, there seems to exist no other "constitutive" property devised so specifically to explain only a certain type of observations (which is also the apparent Q of oscillations). The material Q was originally introduced phenomenologically, by generalizing the observations of creep in rock specimens in the lab [Lomnitz 1957], and supported by mechanical models [Zener 1948], the correspondence principle [Bland 1960], and empirical analogies of seismology with mechanics and acoustics [Knopoff 1964]. In an axiomatic way, the Q was further affirmed in the theory of viscoelasticity (eq. (4)) [Anderson and Archambeau 1964], combined with a hypothesis of functional holomorphism (analyticity) of relations between the *in situ* and observed parameters of the Earth [Anderson *et al*. 1965]. Models of spatially-distributed quality factors were derived for a broad range of frequencies from those of the Chandler wobble and Earth tides [Anderson and Minster 1979] to ultrasonic lab measurements.

Note that the above developments were carried out either by analogy or by mathematical conjecture, but in both cases attempting to present the Q as a primary constitutive parameter of the material. However, as shown below, by systematically considering the problem of energy dissipation in mechanics and solid-state physics, the Q could better be viewed as a *secondary* quantity, derived from the temporal and spatial attenuation (damping) coefficients. Recognition of these attenuation coefficients leads to a considerably simplified observational picture of attenuation [Morozov 2008, 2009a, 2010a] and also to a straightforward theoretical development [Morozov submitted].

2.1. The *Q* and the attenuation coefficient

As illustrated in the physical models below, the inverse Q-factor always arises as a ratio of some temporal attenuation coefficient, χ, to the frequency of oscillation, ω:

$$Q^{-1} \equiv \frac{2\chi}{\omega} . \tag{7}$$

Similarly to χ, this Q is only an apparent (observable) property of the wave. The χ can be frequency independent, but in most cases, it increases with frequency. Note that the basic case of χ = const in eq. (7) leads to $Q^{-1} \propto \omega^{-1}$, which appears to be commonly observed when "elastic" or "geometrical" attenuation (such as scattering or variations of geometrical spreading) are present [Morozov 2008, 2010a, 2011c]. Because of the division by ω in relation (7), Q^{-1} often shows relatively complex frequency dependences, which may include absorption peaks or bands [Anderson *et al*. 1977], or the "high-frequency background" [Cooper 2002].

Our approach is based on analyzing the attenuation coefficient in theory, measurements, and modeling. In the following, we start from the meaning and frequency dependences of χ (and Q) in mechanics and try relating them to the properties of the Earth and planets.

2.2. Energy dissipation in mechanical systems

A wave-propagating medium represents a mechanical system with many degrees of freedom. The most important property characterizing any mechanical system is its Lagrangian function, which often represents a combination of the kinetic and potential energies: $L = E_k - E_p$. In many practical cases, the total mechanical energy, $E_{mech} = E_k + E_p$, is another key function describing the dynamics of the system. Let us now consider a near-stationary process and denote by $\bar{E}_{mech}$ the short-time average (for example, over a period of oscillation) value of E_{mech}. It is commonly observed that in closed systems with weak dissipation, the amplitude of oscillations, $\hat{u}$, exponentially decays with time (Figure 1c):

$$\hat{u}(t) \propto \exp(-\chi t), \tag{8}$$

where χ is the temporal attenuation (damping) coefficient in eq. (7). Note that χ has the dimensions of frequency, and its inverse, $\tau = 1/\chi$, is the “relaxation time” for oscillation amplitude. Broadly, this is the meaning of the empirical relaxation times commonly considered in viscoelasticity [Nowick and Berry1972; Karato and Spetzler 1990].

Relation (8) gives an unambiguous definition for χ; however, it is only available by direct measurements of the amplitude or solving the full equations of motion. In practice, this is rarely possible, and a useful alternative consists in deriving χ from average energy decay rates, which may be measurable more easily. For oscillation (8), the dissipation rate of mechanical energy is proportional to the energy itself:

$$\dot{\bar{E}}_{mech} = -2\chi E_{ref}, \tag{9}$$

where E_{ref} denotes some slowly-varying “reference” measure of mechanical energy. Because it cannot be established which portion of the mechanical energy actually gets dissipated, E_{ref} is a relatively arbitrary level which has to be selected by convention. This level can be the time average of mechanical energy, $E_{ref} = \bar{E}_{mech}$, its peak value $E_{ref} = \hat{E}_{mech}$, or determined in other ways discussed below. Depending on the choices for E_{ref}, the values of χ in (9) may be different, and the “true” value in (8) is obtained by using $E_{ref} = \bar{E}_{mech}$.

Free oscillations

Up to this point, we considered an arbitrary mechanical system, and now let us switch to a single harmonic oscillation at frequency ω_0, such as a standing wave. Such a mode behaves as a damped linear oscillator, which exhibits an amplitude decay in time and a finite-width amplitude peak in the frequency domain (Figure 1). In this case, it is convenient to relate χ to the oscillation frequency, which gives the dimensionless quality factor:

$$Q^{-1} = \frac{2\chi}{\omega_0}. \tag{10}$$

This expression can also be written as:

$$Q^{-1} = \frac{\dot{\bar{E}}_{mech}}{\omega_0 E_{ref}} = \frac{1}{2\pi}\frac{T\dot{\bar{E}}_{mech}}{E_{ref}}. \tag{11}$$

where T is the period of oscillation. At the resonance, E_{mech} is time-independent and represents a natural measure of the reference energy. Because $T\dot{\bar{E}}_{mech}$ equals the mechanical energy dissipated in one period, Q^{-1} is often interpreted as a measure of the relative energy loss in one oscillation cycle. In the frequency domain, this quantity corresponds to the resonator's bandwidth relative to its center frequency ω_0 (Figure 1b):

$$Q^{-1} = \frac{\Delta\omega_{1/2}}{\omega_0}, \tag{12}$$

where $\Delta\omega_{1/2}$ is the width of the spectral peak taken at the level of a half of its power.

The quality factor Q in eqs. (10) – (12) is only defined for systems oscillating near resonance (Figures 1b and c). This parameter describes how strongly damped the resonator is. Resonators with $Q < ½$, $Q = ½$, and $Q > ½$ are called over-damped, critically-damped, or under-damped, respectively, depending on whether they exhibit an oscillatory behavior with time or not (Figures 1c). Dampers keeping doors from slamming shut are examples of critically-damped systems with $Q \approx 1/2$, seismic geophones are usually slightly under-damped, and atomic clocks reach values of $Q > 10^{11}$. For seismic waves, typical Q values range from ~10 to ~5000; however, the meaning of these Q's and their comparisons to (10) and (11) still need to be discussed below.

Energy dissipation is always caused by some type of "internal friction," and in seismology, this term is often equated with Q^{-1} [*e.g.*, Knopoff and MacDonald 1958; Karato and Spetzler, 1990]. Thus, Q^{-1} must be related to certain physical mechanisms of friction. For a damped linear oscillator, friction is caused by the force proportional to the velocity (Figure 1a):

$$F_D = -\zeta \dot{u}, \tag{13}$$

where u is the displacement, and ζ is the viscosity constant. From this relation, the mechanical-energy dissipation rate is always proportional to E_k:

$$-\dot{E}_{mech} = -\dot{u}F_D = \zeta \dot{u}^2 = \frac{2\zeta}{m} E_k, \tag{14}$$

where m is the mass. This is a very important fact that seems to be rarely appreciated: it is the *kinetic energy* that is dissipated by viscous friction, and mechanical-energy loss in a damped linear oscillator is practically unrelated to its elastic "modulus" (*i.e.*, the spring constant $k = m\omega_0^2$).

For time-averaged quantities, eq. (14) reads:

$$-\dot{\bar{E}}_{mech} = \frac{\zeta}{m} \bar{E}_{mech}. \tag{15}$$

because, by virtue of energy equipartitioning, $\bar{E}_{mech} = 2\bar{E}_{kin}$. Again, equations (14) and (15) show that the oscillator's energy simply dissipates *with time* as shown in eq. (9) with $\chi = \zeta/2m$, and *not* with the number of periods as suggested by an intuitive reading of eq. (11). From eq. (10), the oscillator's quality factor (12) equals $Q = m\omega_0/\zeta$.

Thus, for a linear oscillator (or a single oscillation mode in a complex mechanical system), the quality factor is a constant combining the strength of its damper, ζ, with m and ω_0 (Figure 1a). This factor increases with decreasing viscosity and increasing mass, and also with increasing spring constant, k. Linking energy dissipation solely to k is neither necessary nor well-justified from first principles; moreover, this would be opposite to the physical significance of Q as equal $m\omega_0/\zeta$. Nevertheless, such association of attenuation with "imperfect elasticity" is currently common in geophysics [*e.g.*, Anderson and Archambeau 1964; Dahlen and Tromp 1998; Aki and Richards 2002]. In addition, there is no question of frequency dependence of the oscillator's Q, and comparing the Q's at different ω_0's would mean comparing different oscillators.

Forced oscillations

Traveling seismic waves, low-frequency seismic observations in the lab [*e.g.,* Jackson and Paterson 1993], and tidal dissipation (*e.g.,* Efroimsky and Laney 2007) represent forced oscillations at frequencies much lower than the corresponding resonances (if such resonances are present). To describe such phenomena, the notion of Q in eqs. (10) and (11) needs to be extended away from the natural frequencies. However, for forced oscillations, the energy is not equipartitioned, and E_{mech} no longer uniquely describes the state of the system. In the right-hand sides of eqs. (10) and (11), various measures of energy can be used, leading to several types of Q. For example, eq. (14) suggests that the natural choice for E_{ref} could be the peak *kinetic* energy, $E_{ref} = \hat{E}_k/2$ (denominator 2 here is because of using the peak amplitude of $E_k(t) = \hat{E}_k(1+\cos 2\omega t)/2$ as a measure of its average level). Therefore, from eq. (11), the "kinetic" quality factor Q_k is proportional to frequency (Figure 2):

$$Q_k = \frac{\omega \hat{E}_k}{2\dot{\bar{E}}_{mech}} = \frac{\omega}{\omega_0} Q \,. \tag{16}$$

Alternately, if we relate the dissipation to the peak *potential* energy, $E_{ref} = \hat{E}_p/2$, as it is done in seismology [Aki and Richards 2002], then such quality factor becomes inversely proportional to frequency (Figure 2):

$$Q_p = \frac{\omega \hat{E}_p}{2\dot{\bar{E}}_{mech}} = \frac{\hat{E}_p}{\hat{E}_k} Q_k = \frac{\omega_0}{\omega} Q \,, \tag{17}$$

because $\hat{E}_p = (\omega_0/\omega)^2 \hat{E}_k$ in a forced oscillation. Next, if we define the quality factor Q_t from the relative loss of the *peak total* mechanical energy, then:

$$Q_t = \frac{\omega \hat{E}_{mech}}{\dot{\bar{E}}_{mech}} = \begin{cases} Q_p \text{ for } \omega < \omega_0, \\ Q_k \text{ for } \omega \geq \omega_0, \end{cases} \tag{18}$$

since $\hat{E}_{mech}$ equals $\hat{E}_p$ or $\hat{E}_k$ for $\omega < \omega_0$ and $\omega \geq \omega_0$, respectively (Figure 2).

Further, note that E_{ref} in definition (11) should be best understood as representing the stored energy *averaged* over a period. Peak energies, which are attained only once or twice during a period, should unlikely have any definite relation with the average $\dot{\bar{E}}_{mech}$. At resonance, $\bar{E}_{mech} = \hat{E}_{mech}$, and for $\omega \neq \omega_0$, the energy oscillates between $\hat{E}_k$ and $\hat{E}_p$ above, and its average level equals $\left(\hat{E}_k + \hat{E}_p\right)/2$. Consequently, we can define the "average-total energy Q" as:

$$Q_a = \frac{\omega \bar{E}_{mech}}{\dot{\bar{E}}_{mech}} = \frac{Q_k + Q_p}{2}. \tag{19}$$

This seems to be the most logical choice for the oscillator's Q away from resonance. Note that it stays near constant for $\omega \approx \omega_0$ (Figure 2).

Yet another definition of Q comes from force-displacement phase lags δ in sub-resonant lab measurements of seismic attenuation [*e.g.,* Lomnitz 1957; Jackson and Paterson 1993] and observations of Earth's tides [Efroimsky and Laney 2007; Agnew 2009]. If we consider a harmonic force $f(t) = \hat{f}e^{-i\omega t}$ and a logarithmic amplitude decay rate χ, the stationary solution for displacement is (Figure 1b):

$$u(\omega) = \frac{\hat{f}}{\omega_0^2 - 2i\chi\omega - \omega^2}. \tag{20}$$

This displacement lags the force by phase δ:

$$\delta = Arg\left(\omega_0^2 - \omega^2 + 2i\chi\omega\right) = \tan^{-1}\frac{2\chi\omega}{\omega_0^2 - \omega^2}. \tag{21}$$

At low frequencies $\omega \ll \omega_0$, cotangent of the phase lag δ is usually interpreted as the quality factor in lab and tidal measurements: $Q_\delta \equiv 1/\tan\delta$ [*e.g.,* Jackson and Paterson 1993; Efroimsky and Laney 2007]. This quantity approximates the quality factors Q_p and Q_t above, and is inversely proportional to the frequency (Figure 2):

$$Q_\delta = \frac{\omega_0^2 - \omega^2}{2\chi\omega} \approx \frac{\omega_0^2}{2\chi\omega} = Q_p. \tag{22}$$

As we see, several types of forced-oscillation Q factors can be defined for a linear oscillator, based on somewhat different physical ideas and approximations. All of these definitions contain strong and variable frequency dependencies (Figure 2). These extensions of the "true" resonance Q away from the natural frequency characterize not so much the oscillator itself but mostly our preference for selecting the reference energy. Therefore, when applied to rocks and wave processes in different environments, the "quality factors" (10), (11), and (16)–(19) should be compared to each other very carefully, because they may represent quite different physical characteristics of the system.

2.3. Energy dissipation in seismology and geodesy

The seismological Q is usually introduced as follows [*e.g.,* Aki and Richards 2002, pp. 162–163). If a volume of Earth's material is cycled in stress at circular frequency ω, such as caused by a traveling or standing wave, then the energy loss in one cycle, $\delta E = T\dot{\bar{E}}_{mech}$, is proportional to the peak strain energy, $\hat{E}_{strain}$, stored in that volume. A dimensionless ratio of these quantities is formed and denoted Q^{-1}:

$$Q^{-1} = \frac{1}{2\pi}\frac{\delta E}{\hat{E}_{strain}} = -\frac{\dot{\bar{E}}_{mech}}{\omega E_{strain}}. \tag{23}$$

The same definition is adopted for tidal Q [Greenberg 2009]. By its use of the elastic energy for reference ($E_{ref} = E_{strain}$), this expression corresponds to our mechanical Q_p in eq. (17).

It is important to see that maybe contrary to its intended purpose, the Q^{-1} defined by eq. (23) is an attribute of the entire oscillation (wave) rather than a local property of the medium. If the "volume" mentioned in the above definition represents the entire oscillating body, then eq. (23) is equivalent to those discussed in section 2, and this Q^{-1} is a well-defined averaged quantity. However, if the "volume" is considered as (macroscopically) small, then ratios (23) taken at different points should differ even though the material may be the same. The energy dissipation rate depends on a number of different factors, for example, $\dot{E}_{mech} = -\int \frac{\partial D}{\partial \dot{\varepsilon}_{ij}} \dot{\varepsilon}_{ij} dV$ for solid viscosity, $\dot{E}_{\mathrm{mech}} = -\frac{\kappa}{T}\int \left(\vec{\nabla} T\right)^2 dV$ for heat conduction [Landau and Lifshitz 1986], and $\dot{E}_{mech} \propto -\int u^2 r^2 dV$ for random reflectivity [Morozov 2011c]. In these expressions, D is the dissipation function, $\dot{\varepsilon}_{ij}$ is the strain-rate tensor, T is the temperature, κ is the thermal conductivity, u is the displacement amplitude, and r is the reflection coefficient. These quantities depend on deformation amplitudes but none of them are closely related to E_{strain}, and therefore ratios (23) should vary for adjacent points in the same material.

Quantity (23) is measured from the energy dissipation rate from a fixed volume within the medium, and therefore this is the *temporal* Q^{-1}, related to the temporal attenuation coefficient χ as (compare to (4) and (7)) [Morozov 2010]:

$$Q_{temporal}^{-1} = \frac{2\chi}{\omega}. \tag{24}$$

For a traveling wave, the oscillations are often stationary and energy dissipation is manifested by a spatial amplitude decay. Therefore, the corresponding spatial Q^{-1} is:

$$Q_{spatial}^{-1} = \frac{2\alpha}{k}, \tag{25}$$

where k is the wavenumber and α is the spatial attenuation coefficient [Aki and Richards 2002]. Similarly to (23)–(24), this is a property of the entire wave. Because it refers to the spatial harmonic k, it is clearly not localized in space.

In summary, ratios (23)–(25) principally depend on the distributions of amplitudes, temperatures, strains and strain rates within the body, and they are not very suitable for describing its material. Perhaps the best way of treating such quantities, along with the Q's (17)–(19) and (22), is to take them as "figures of merit" [Armstrong 1980], or apparent quantities describing the wave processes. Because of this lack of physical basis for the notion of material Q (and also because of the way they are produced by viscoelastic inversions), it is difficult to judge about the physical significance of, for example, the existing models of spatially-variable Q within the Earth (such as PREM). Such a relation might come, for example, from relating the Q's to mantle viscosities and thermoelastic parameters. However, this would amount in refitting the raw attenuation data and could be a subject of further studies [Morozov unpublished].

2.4. Practical issues

Because of the two ways the material quality factor is defined (empirically in observations and axiomatically in the viscoelastic model), this parameter also leads to several practical problems. As an empirical quantity, the Q trades off with geometrical spreading and depends on the assumptions involved in measurement procedures, modeling and inversion. Some debate about these problems was carried out recently [Morozov 2008, 2009a, b; Xie and Fehler 2009; Morozov 2010], with focus on the geometrical spreading and the frequency dependence of Q [Mitchell 2010; Xie 2010]. As we pointed out from several datasets [Morozov 2008, 2010, b, 2011b], improved corrections for geometrical spreading often eliminate the frequency dependence of Q and increase its values at 1 Hz (often denoted Q_0) by up to 20–30 times. However, the problem is actually much deeper than the frequency dependence of Q. For example, another major difficulty can be seen from the fact that in physical models [*e.g.*, Müller 2010], wave Q^{-1} increases with viscosity and decreases with temperature, whereas in seismology, this correlation is opposite [Sato 1991].

Serious practical issues also arise from the axiomatic definition of Q in the viscoelastic theory (eq. (4)). For example, when $\mathrm{Re}M = 0$, this Q^{-1} is undefined, and the whole viscoelastic model of attenuation does not work. In consequence, in all global Q models of the Earth [such as PREM, Dziewonski and Anderson 1981], shear dissipation within the outer core is set equal zero: $Q_\mu^{-1} = 0$. Nevertheless, the outer core possesses viscosity and substantial shear deformations (for example, in free oscillations), and therefore it clearly dissipates the shear energy.

3. Two thought experiments

Similarly to the case of damped linear oscillator in section 2.1, physical models of anelastic materials often show that energy dissipation is caused by their internal movements and not only by deformation [*e.g.*, Biot 1956]. This again contradicts the assumed definition of seismic Q^{-1} in (23), which can be seen from a simple thought experiment (Figure 3). Consider a small specimen of "wet" porous rock welded into a tight, infinitely rigid casing, so that it does not deform, and consequently its $\hat{E}_{strain} = 0$ for any form of the viscoelastic moduli. Let us now assume that the casing is subjected to some movement by an external force (Figure 3). Despite the absence of macroscopic deformation, the mechanical energy should still dissipate within the specimen due to internal pore-fluid movements. This would lead to unreasonable $Q = 0$ from eq. (23).

Another thought example shows that the "material memory" and Q actually depend on the way the experiment is conducted (Figure 4). Consider a specimen of perfectly elastic material subjected to cyclic creep testing. First, let the specimen be thermally insulated during the initial compression or expansion (stages AC or BD in Figure 4a). Because of the effect of thermal expansion, the temperature of the specimen will change (increase in AC and decrease in BD), and we will allow it to equilibrate at constant stresses during stages CB and DA (Figure 4a). In this case, despite the perfect elasticity of the specimen, mechanical energy would still be dissipated, as indicated by the area of the hysteresis loop ACBDA, and the resulting Q^{-1} would be positive (Figure 4a). Of course, in a more realistic case of incomplete insulation, thermal relaxation would proceed concurrently with deformation, and the hysteresis loop would have a shape closer to the one shown by the dashed line in Figure 4a.

By contrast to the case in Figure 4a, if the temperature of the specimen is maintained constant, the specimen would respond with the isothermal modulus [Hayden *et al*. 1965] and deform from A to B and back reversibly (Figure 4b). The net energy dissipation would then equal zero, and $Q^{-1} = 0$ (Figure 4b). This shows that the Q measured in lab creep tests (as well as in all other situations) actually depends on how the heat is supplied or removed from the various parts of the body.

The above examples show that energy dissipation is not always (if ever) related to the "imperfect" elastic moduli. Many types of energy dissipation *cannot* be accounted for by the viscoelastic moduli with fading memory. If such moduli are still used phenomenologically, they become dependent on the experimental environments and may be tricky to interpret. Furthermore, the fading memory should depend on the parameters of experiments, such as the shapes and dimensions of the specimens. This is difficult to illustrate on an intuitive thought example, and a more theoretical argument is given in the next section.

One can argue that the viscoelastic model is only intended for relatively uniform, "dry" materials which do not exhibit thermoelastic effects. However, such cases are of hardly any significance in geophysics. The rocks comprising the Earth exhibit significant thermal expansion, they are generally grainy and porous, and pore fluids and melts are well known as important contributors to both seismic velocities and attenuation. The dimensions of rock samples sometimes comprise only ~5% of those of the attenuation measurement apparatuses [Gribb and Cooper 1998], and therefore the spatial heterogeneity is the first-order factor. Spatial gradients, heterogeneity, and wave processes are also the most important phenomena studied in observational seismology.

4. Viscoelastic approximation

4.1. Character of viscoelastic approximation

In viscoelasticity, the complexity of dissipation mechanisms is replaced with a phenomenological strain-stress relation within the deformed material. All interactions within the medium are expressed as a time-dependent stress-strain relation applied at each point [*e.g.,* Dahlen and Tromp 1998]:

$$\sigma\left(t\right)=\int_{-\infty}^{t}M'\left(t-\tau\right)\dot{\varepsilon}\left(\tau\right)d\tau\,, \tag{26}$$

where the convolutional kernel $M'(\tau)$ is related to M in eq. (3) by $M=\dot{M}'$. This general form representing the stress as a linear combination of the preceding strain rates is known as the Boltzmann's superposition principle. In materials-science literature [*e.g.,* Nowick and Berry 1972; Cooper 2002], this relation is often reversed, giving the strain responding to stress rate:

$$\varepsilon\left(t\right)=\int_{-\infty}^{t}J\left(t-\tau\right)\dot{\sigma}\left(\tau\right)d\tau\,, \tag{27}$$

where $J(\tau)$ is the time-retarded compliance. Functions $M(t)$, $M'(\tau)$, $J(t)$ and/or their frequency-domain counterparts are constructed so that they lead to the observed creep curves, phase velocities and apparent Q^{-1}spectra for the deformations of interest. Such functions can be found by combining multiple dissipation mechanisms operating at different frequencies. Hayden *et al.* [1965], Nowick and Berry [1972], and Karato and Spetzler [1990] gave examples of such mechanisms, generally related to several kinetic processes taking place within the material under stress. Each of these processes can be phenomenologically described as a response of some "equivalent mechanical system," such as the standard linear solid [Zener 1948]. Liu *et al.* [1976] modeled the general forms of relations (26)–(27) by multiple standard linear solids connected in parallel (Figure 5).

Although integral expressions (26)–(27) are able to describe rock-creep observations in the lab, they are nevertheless unsatisfactory as a constitutive law, because they contradict the common observations of perturbations always spreading spatially in solids and fluids. It is important to clearly understand whether these equations describe the behavior of a *material* or a finite *body* (rock specimen) in a lab experiment. In reality, near-linear relations between ε and σ similar to (26) result from interactions within the body. Time-delayed interactions do not occur at single points but result from deformations, heat flows and waves spreadng through the body and affected by its boundaries and heterogeneities. Therefore, the apparent "memory" associated with a point largely represents the structure of the surrounding body. A more general form of the Boltzmann's superposition principle valid for heterogeneous and wave-propagating media should be:

$$\sigma\left(\mathbf{r},t\right)=\int d^3\mathbf{r}'\int_{-\infty}^{t} M\left(\mathbf{r},\mathbf{r}',t-\tau\right)\varepsilon\left(\mathbf{r}',\tau\right)d\tau\ , \tag{28}$$

where $M(\mathbf{r},\mathbf{r}',\tau)$ is the (tensor) impulse response of the stress at point $\mathbf{r}$ to strain at point $\mathbf{r}'$ at time τ in the past. To ensure causality, this response must equal zero for τ exceeding the travel time of the fastest wave from point $\mathbf{r}'$ to $\mathbf{r}$. At the same time, M is not proportional to $\delta(\mathbf{r}-\mathbf{r}')$, as implied in eq. (26), and we cannot interpret this quantity as a "modulus" of the medium. Quantity $M(\mathbf{r},\mathbf{r}',\tau)$ in eq. (28) is merely a point-source solution to the equations of motion for the deformed body.

The key aspect of the viscoelastic approximation consists in presenting the *thermodynamic properties* of the medium as *time variations* of the moduli in the empirical integral (26). For example, for deformations conducted at constant temperatures, the medium responds with an isothermal modulus (k_I), and for adiabatic processes, the corresponding modulus is $k_A \geq k_I$ (Figure 6) [Hayden *et al.* 1965]. The easiest way (but by no means the only one) to implement such regimes in the lab is by using very slow and fast deformations, respectively. In viscoelasticity, exactly such type of experiment is implied, and consequently the modulus k_A is called "unrelaxed" (*i.e.* acting at time scales $t \to 0$) and denoted k_U, and modulus k_I is called "relaxed," k_R, presumably acting in the limit $t \to \infty$ (Figure 5) [Nowick and Berry 1972, p. 8]. However, in reality, the difference between these two moduli is thermodynamic and has nothing to do with the time scales. For example, $k_A = k_U$ would operate even in "slow" processes with thermal insulation (Figure 4a), and the isothermal behavior $k_I = k_R$ can be achieved without any relaxation, by controlling the temperature of the body (Figure 4b). The pair (k_U, k_R) only applies to some specific thermal regime during deformation. Note that this thermal regime is usually not specified when moduli k_U and k_R are discussed [Nowick and Berry 1972].

The σ-to-ε relations in viscoelastic systems (26)–(27) are often explained by using arrangements of springs and dashpots, such as the one shown in Figure 5. Although it is generally understood that such models have generally only a pedagogical role for non-specialists [Lakes 1999, p. 23], they still dominate the thinking of many materials experimentalists and theoreticians [Cooper 2002]. For our discussion, note that such systems *are completely mechanical*, and contrary to what was stated in the *Introduction*, it may therefore appear that viscoelasticity does not contradict the mechanical framework. However, it is important to see how such mechanical behavior is achieved.

All mechanical models which successfully predict creep and different relaxed and "unrelaxed" moduli of viscoelastic solids (*i.e.*, the Maxwell, standard linear, or Burgers solids) always contain internal variables, such as the one indicated by ξ in Figure 5. Such variables are critical for these models, because they are directly responsible for creep. The Hookean stress-strain relation $\sigma(\varepsilon) = k\varepsilon$ is modified by the internal variables; for example, for the standard linear solid (dashed box in Figure 5) it becomes:

$$\sigma(\varepsilon, \xi) = (k_R + k_1)\varepsilon - k_1\xi \,. \tag{29}$$

At the same time, the models contain no masses (*i.e.*, zero kinetic energy) associated with such internal variables (dots in Figure 5). This massless character of internal variables is critical for making them behave kinetically:

$$\dot{\xi} = -\frac{k_1}{\eta}\xi + \frac{k_1}{\eta}\varepsilon \,, \tag{30}$$

which allows solving for their time dependences $\xi(t)$ as Volterra integrals of $\varepsilon(t)$ or $\varepsilon(t)$ in eqs. (26)–(27). However, massless internal variables are quite artificial, and physical models [*e.g.*, Biot 1956] usually show that internal variables do contribute to the kinetic energy. Ignoring the kinetic energy associated with ξ again shows that the deformation in Figure 5 should be slow, *i.e.*, *quasi-static*. Also, in real continuous systems, perturbations of the internal variables should often spread spatially (for example, diffuse) and not just follow the kinetic equation (30) at the individual points. Therefore, mechanical models of viscoelastic bodies also assume *quasi-uniform* deformations.

Thus, viscoelasticity represents a rather special limit of the solid-state mechanics for quasi-static, uniform-medium interactions, with mechanisms of anelasticity limited to kinetic phenomena, and thermodynamic relations replaced with time-scale phenomenologies. Arguably, this approximation may be suitable for experimental materials science and chemical kinetics, in which only responses of uniform media to near-constant stresses are usually discussed [Nowick and Berry 1972; Cooper 2002]. For such problems, equivalent mechanical models (*e.g.*, Figure 5) provide convenient pictorial descriptions without involving the intricate formalism of theoretical physics. Practically the same quantity, namely Q^{-1}, is attributed to both the *in situ* and observational levels, and thereby the theory becomes greatly simplified. The frequency (and temperature) dependence of the *in situ* Q provides a very flexible parameterization which allows fitting and modeling the observed creep or Q^{-1} data. Because of the spatial uniformity, this phenomenological Q can be qualitatively related to some kinetic processes within the material [Karato and Spetzler 1990; Cooper 2002].

However, in cases of spatial heterogeneity, such as considered in seismology and geodesy, the above approximation is inadequate. Treating the *solution of the equations of motion* (28) in some body as a time-delayed *constitutive law* (26) at each point $\mathbf{r} = \mathbf{r}'$ within the medium and replacing the thermodynamic effects by empirical time-relaxation laws may be grossly misleading. This makes the resulting $M'(\mathbf{r},\omega)$ and $Q(\mathbf{r},\omega)$ in (26) only a heuristic mathematical parameterization.

3.2. Material *Q* and the correspondence principle

The correspondence principle is the theoretical foundation of viscoelasticity and of its concept of material *Q* [*e.g.*, Bland 1960; Aki and Richards 2002]. This principle states that the elastic-energy dissipation within a material can be described by replacing the wave speed *V* with a complex-valued V' (compare to eq. (5)):

$$V' = V\left(1 - \frac{i}{2Q}\right). \tag{31}$$

This principle automatically attributes a *Q* to each macroscopic point within the medium. Furthermore, because the *in situ* Q^{-1} becomes interpreted as a part of velocity, forward modeling and inversion methods for *V* (for example, 3-D tomography) become readily applicable to $VQ^{-1} = -2\,\mathrm{Im}\,V'$ [*e.g.*, Anderson *et al*., 1965; Romanowicz and Mitchell 2009].

Nevertheless, the above similarity of VQ^{-1} and $V = \mathrm{Re}\,V'$ actually suggests a problem with interpreting relation (31). Anelasticity is vastly different from wave velocity, both by its effect on the wave and by the characters and by the variety of physical mechanisms involved. The (perhaps rarely noticed) subtlety consists in the fact that the velocity *V* in eq. (31) is the *phase* velocity, which is often dispersive and generally (in heterogeneous media) different from the wave speed. Therefore, Q^{-1} is contained in the imaginary part of the phase velocity, which can also be seen from eq. (25) above. This is the *apparent* Q^{-1} of the wave. The wave speed, which is a physical property of the medium, remains real-valued and frequency-independent. Therefore the correspondence principle does not help in elucidating the meaning of the material *Q*. For more discussions of the correspondence principle in heterogeneous media, see Morozov [2011a].

5. Discussion

The following argument is often advanced[1] against the above critique of the *in situ Q*: if the material *Q* cannot be considered a valid local property of the medium, then a similar argument can be made about the seismic velocity, *V*. However, *V* is a broadly accepted and important property of the wave-propagating medium, and therefore the critique of *Q* must somehow be flawed.

Nevertheless, the above inference regarding *V* is actually correct, but we need to clearly understand what type of "seismic velocity" is being meant here. As explained in the preceding section, the velocity associated with a *Q* is the *phase velocity*, V_{phase}, which is attributed to the entire propagating wave and is not distributed in space. Together with *Q*, V_{phase} in fact only represents the complex wavenumber:

$$k' = \frac{\omega}{V_{\text{phase}}}\left(1 + \frac{i}{2Q}\right), \tag{32}$$

which describes the spatial variation of wave amplitudes:

$$u(x,t) = \hat{u}\exp\left[-\left(\mathrm{Im}\,k'\right)x\right]\exp\left[-i\omega t + i\left(\mathrm{Re}\,k'\right)x\right]. \tag{33}$$

This *de facto* variation of the wave vector can be caused by any physical reasons, and its complex

[1] Unfortunately, by anonymous reviewers.

argument, $Arg\left(\left|\mathbf{k}'\right|\right)$, hardly has any unique significance. The wave vector, $\mathbf{k}'$, is clearly non-local, and so are V_{phase} and Q. By contrast, the *wave speed* of the medium is a combination of its mechanical parameters and a local property, such as $V_S = \sqrt{2\mu/\rho}$ for S waves. In a heterogeneous medium (for example, for a surface wave), V_{phase} practically nowhere equals the wave speed. Thus, there is actually no reason to believe that V_S and μ attain imaginary parts and become frequency-dependent in anelastic media.

If questioning the physical significance of the *in situ Q* and the usefulness of the viscoelastic theory in seismology, how can we describe the seismic or tidal attenuation? Unfortunately, no universal answer to this question seems to exist. Many factors influence mechanical-energy dissipation within the Earth, and they are poorly understood at present. For isotropic, porous, fluid-saturated sedimentary rock, Biot's [1956] framework presented numerous insights, which were extended, for example, to Rayleigh waves by Deresiewicz [1960]. For recent reviews of poroelasticity, see Müller et al. [2010] and Singh [2011]. However, for deep crustal and mantle conditions, the physical mechanisms of energy dissipation seem unclear at present. Despite the predominance of the idea that Earth's anelasticity is mostly due to kinetic ("microdynamic") processes occurring at the scale of material grain sizes, such as the diffusion of dislocation and point defects [*e.g.*, Nowick and Berry 1972; Karato and Spetzler 1990; Cooper 2002; Romanowicz and Mitchell 2009], it appears that solid viscosity [Landau and Lifshitz 1986] and thermoelastic phenomena [Hayden *et al*. 1965] may produce comparable, if not much stronger, contributors to it. Such phenomena are sensitive to the structural heterogeneity at the scale lengths of 0.1–500 m [Morozov *submitted*]. This shows that the Earth's structure itself represents a significant factor in explaining the anelastic effects. In other words, the "grain size" which is effective for energy dissipation may be much larger than the mineral-aggregate granularity commonly studied in the lab and inferred for the mantle. The importance of such factors may be most apparent from the seismic observations on the Moon. Nakamura and Kayama [1982] noted that the somewhat unusual properties of seismic attenuation on the Moon may be due to the thermoelastic effects within its crust. Qualitatively, this agrees with the observations of very strong scattering and interpretations of pervasive heterogeneity of the lunar crust [Toksöz *et al*. 1974]. However, this subject requires an extensive further research.

6. Conclusions

The current viscoelastic model represents a phenomenological, quasi-static approximation to the full physical description of anelasticity in Earth materials. This approximation can arguably be used for describing the behavior of rock samples in lab experiments; however, it should generally be inadequate for seismology or whole-Earth deformations.

The viscoelastic concepts of "material memory" and "material *Q*" in fact only describe the history of deformation for a given body. Both of these properties are controlled by the experimental environments and should not be viewed as constitutive attributes of the material. In particular, the process of relaxation is determined not by the time extent of stress application (as often thought) but by the thermodynamic conditions of deformation. Several important types of mechanical-energy dissipation cannot be accounted for by the material memory at all; for example: 1) wet rock can dissipate energy without macroscopic deformation, 2) fluid zones dissipate shear energy despite their shear modulus being equal zero, and 3) a perfectly elastic rock also dissipates mechanical energy in spatially-heterogeneous environments.

The *Q* is hardly valid physically when viewed as a local property of the medium and related to the viscoelastic moduli. By using several "reasonable" analogies with mechanics, multiple definitions of seismic *Q* can be obtained, all with different frequency dependences. This shows that the material quality factor is largely controlled by the assumptions implied in its definition.

In contrast to the limitations of the viscoelastic model, rigorous physical approaches to macroscopic solid and fluid continua are well known and successfully account for all aspects of anelasticity. These approaches neither require nor lead to the material memory or *Q* and reveal the true mechanisms of seismic and tidal attenuation. Broadly, such mechanisms include: 1) solid and pore-fluid viscosity (rheology), 2) thermoelasticity, 3) various kinds of kinetic processes, and 4) effects of the Earth's structure. These mechanisms operate at all time scales and conditions, and their better understanding would lead to a unified picture of seismic and geodetic anelasticity.

Acknowledgments

This research was supported by Canada NSERC Discovery Grant RGPIN261610-03.

References

Agnew, D.C. (2009). Earth tides, *in*: Romanowicz, B., and A. Dziewonski (Eds.), Treatise on Geophysics, v. 3, Geodesy, ISBN 978-0-444-53460-6, p. 163–195.

Aki, K. and P.G. Richards (2002). Quantitative Seismology, Second Ed., University Science Books, Sausalito, California.

Anderson, D.L. and C.B., Archambeau (1964), The anelasticity of the Earth, J. Geophys. Res., 69, 2071–2084.

Anderson, D.L., A. Ben-Menahem, and C.B. Archambeau, (1965). Attenuation of seismic energy in the upper mantle, J. Geophys. Res., 70, 1441-1448.

Anderson, D.L., H. Kanamori, R.S. Hart, and H.-P. Liu (1977), The Earth as a seismic absorption band, Science, 196 (4294), 1104 – 1106.

Andrade, E.N. da C. (1910), On the viscous flow in metals, and allied phenomena, Proc. R. Soc. Lond. A, 84, 1–12

Armstrong, B.H., 1980. Frequency-independent background internal friction in heterogeneous solids, *Geophysics* 1042–1054.

Biot, M.A. (1956). Theory of propagation of elastic waves in a fluid-saturated porous solid. J. Acoust. Soc. Am. 28, (a) 168–178, (b) 179–191.

Bland, D. (1960), The theory of linear viscoelasticity, Pergamon Press.

Cooper, R. (2002). Seismic wave attenuation: Energy dissipation in viscoelastic crystalline solids, in: S.-I, Karato and H. R. Wenk, eds., Plastic deformation of minerals and rocks, Rev. Mineral Geochem., 51, 253–290.

Cormier, V.F. (2011). Seismic viscoelastic attenuation, *in*: Gupta, H. K. (Ed.), Encyclopaedia of Solid Earth Geophysics: Springer, doi: 10.1007/978-90-481-8702-7, 1279–1289.

Gribb, T.T., and R.F. Cooper, (1998). Low-frequency shear attenuation in polycrystalline olivine: Grain boundary diffusion and the physical significance of the Andrade model for viscoelastic rheology, J. Geophys. Res., 103, 27267–27279.

Dahlen, F.A. and J. Tromp (1998), Theoretical global seismology. Princeton Univ. Press, Princeton, NJ.

Deresiewicz, H. (1960). The effect of boundaries on wave propagation in a liquid-filled porous solid: I. Reflection of plane waves at a free plane boundary (non-dissipative case), Bull. Seism. Soc. Am., 50, 599–607.

Dziewonski, A.M., and D.L. Anderson (1981). Preliminary Reference Earth Model (PREM), Phys. Earth Planet. Inter., 25, 297–356.

Efroimsky, M., and V. Laney (2007). On the theory of bodily tides, *in*: Belbruno, E. (Ed.) New trends in Astrodynamics and Applications III, American Institute of Physics Conference proceedings, vol. 886, ISBN 978-0-7354-0389-5, p 131–138.

Fabrizio, M. and A. Morro (1992), *Mathematical problems in linear viscoelasticity*, SIAM studies in applied mathematics, v. 12, ISBN 0-89871-266-1

Greenberg, R. (2009). Frequency dependence of tidal Q, Astron. J., 698, L42–L45.

Hayden, H.W., W.G. Moffatt, and J.M. Wulff (1965), Structure and properties of materials, Volume III. Mechanical behavior, John Wiley & Sons.

Jackson, I., and M.S. Paterson (1993). A high-pressure, high-temperature apparatus for studies of seismic wave dispersion and attenuation, Pure Appl. Geoph., 141 (2/3/4), 445–466.

Jackson, I., S. Webb, L. Weston, and D. Boness (2005). Frequency dependence of elastic wave speeds at high temperature: a direct experimental demonstration, Phys. Earth Planet. Inter., 148, 85–96.

Karato, S., and H.A. Spetzler, (1990). Defect microdynamics in minerals and solid-state mechanisms of seismic wave attenuation and velocity dispersion in the mantle, Rev. Geophys., 28, 399–421.

Karato, S.-I. (2008). Deformation of Earth materials. An introduction to the rheology of Earth materials, Cambridge Univ. Press, ISBN 978-0-521-84404-8

Karato, S.-I., and P. Wu (1993), *Rheology of the upper mantle*: A synthesis, *Science*, *260*, 771–778.

Knopoff, L. (1964). *Q*. Rev Geophys., 2, 625–660.

Knopoff, L., and G.J.F. MacDonald (1958), Attenuation of small amplitude stress waves in solids, Rev. Mod. Phys., 30, 1178–1192.

Landau, L.D., and E.M. Lifshitz (1976). Course of theoretical physics, Volume 1: Mechanics. Third edition, Butterworth-Heinemann, ISBN 0 7506 2896 O

Landau, L.D., and E.M. Lifshitz (1986). Course of theoretical physics, volume 7 (3rd English edition): *Theory of elasticity*, Butterworth-Heinemann, ISBN 978-0-7506-2633-0

Landau, L.D., and E.M. Lifshitz (1987). Course of theoretical physics, volume 6: Fluid mechanics, (2nd English edition): Butterworth-Heinemann, ISBN 978-0-7506-2767-2

Lakes, R.S. (1999). *Viscoelastic solids*, CRC Press, Boca Raton, FL.

Lifshitz, E.M., and LP. Pitaevskii, (1981). *Physical kinetics*, Landau and Lifshitz Course of theoretical physics, volume 10, Butterworth-Heinemann, ISBN 978-0-7506-2635-4

Liu, H.P., D.L. Anderson, and H. Kanamori (1976). Velocity dispersion due to anelasticity: implications for seismology and mantle composition, *Geophys. J. R. Astr. Soc.*, *47*, 41–58.

Lomnitz, C. (1957). Linear dissipation in solids, *J. Appl. Phys.*, *28*, 201–205.

Mitchell, B. (2010). Prologue and invitation to participate in a forum on the frequency dependence of seismic *Q*, Pure Appl. Geophys. 167, 1129, doi 10.1007/s00024-010-0180-3

Morozov, I.B. (2008). Geometrical attenuation, frequency dependence of *Q*, and the absorption band problem, Geophys. J. Int., 175, 239–252.

Morozov, I.B. (2009a). Thirty years of confusion around "scattering *Q*"? Seism. Res. Lett., 80, 5–7.

Morozov, I.B. (2009b). Reply to "Comment on 'Thirty years of confusion around 'scattering *Q*'?" by J. Xie and M. Fehler, *Seism. Res. Lett.*, *80*, 648–649.

Morozov, I.B. (2010). On the causes of frequency-dependent apparent seismological *Q*. *Pure Appl. Geophys.*, 167, 1131–1146, doi 10.1007/s00024-010-0100-6.

Morozov, I.B. (2011a), Anelastic acoustic impedance and the correspondence principle. *Geophys. Prosp.,* doi 10.1111/j.1365-2478.2010.00890.x.

Morozov, I.B. (2011b). Temporal variations of coda *Q*: An attenuation-coefficient view, *Phys. Earth Planet. Int.*, *187*, 47–55

Morozov, I.B. (2011c). Mechanisms of geometrical seismic attenuation, Annals of *Geophysics*, *54*, 3, 235–248.

Morozov, I.B. (submitted to J. Geophys. Res.). Mechanics of tidal and seismic energy dissipation. I: Fundamentals, http://seisweb.usask.ca/ibm/papers/Q/Mechanics_2011_part1.preprint.pdf, accessed 12 Sept 2011, accessed 6 Oct 2011.

Morozov, I.B. (unpublished), Viscosity model for the Earth from free oscillations, http://seisweb.usask.ca/ibm/papers/Q/Morozov_Free_Oscillations.preprint.pdf, accessed 12 Sept 2011.

Morozov, I.B., C. Zhang, J.N. Duenow, E.A. Morozova, and S. Smithson (2008). Frequency dependence of regional coda *Q*: Part I. Numerical modeling and an example from Peaceful Nuclear Explosions, Bull. Seism. Soc. Am., 98, 2615–2628, doi: 10.1785/0120080037

Müller, T., B. Gurevich, and M. Lebedev (2010). Seismic wave attenuation and dispersion resulting from wave-induced flow in porous rock – A review, *Geophysics*, *75*, 75A147 – 75A164.

Nakamura, Y., and J. Koyama (1982), Seismic *Q* of the lunar upper mantle, *J. Geophys. Res.*, *87*, 4855–4861

Nowick, A.S., and B.S. Berry (1972*). Anelastic relaxation in crystalline solids*, Academic Press, 677 pp.

Romanowicz, B. and B.J. Mitchell (2009). Deep Earth structure – *Q* of the Earth from crust to core, *in*: Romanowicz, B., and A. Dziewonski (Eds.) Treatise on Geophysics, v. 1, Seismology and Structure of the Earth, ISBN 978-0-444-53459-0, p. 731–774.

Sato, H. (1991). Viscosity of the upper mantle from laboratory creep and anelasticity measurements in peridotite at high pressure and temperature, Geophys. J. Int., 105, 587–599.

Singh, B. (2011). On propagation of plane waves in a generalized porothermoelasticity, Bull. Seism. Soc. Am. 101, 765–762, doi: 10.1785/0120100091

Toksöz, M.N., A.M. Dainty, S.C. Solomon, and K.R. Anderson (1974). Structure of the Moon, Rev. Geophys. Space Phys., 12, 539–567.

Xie, J. (2010). Can we improve estimates of seismological *Q* using a new "geometrical spreading" model? Pure Appl. Geophys., 167, 1147–1162, doi: 10.1007/s00024-010-0188-8

Xie, J. and M. Fehler (2009). Comment on "Thirty years of confusion around scattering *Q*" by Igor. B. Morozov, Seism. Res. Lett., 80, 646–647.

Zener, C. M. (1948). Elasticity and Anelasticity of Metals, University of Chicago Press.

Figures

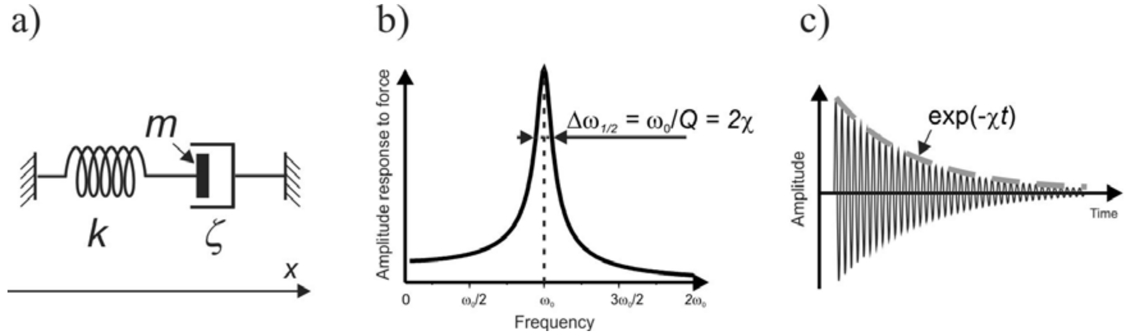

Figure 1. Mechanical-energy dissipation: a) damped linear oscillator, b) its response to harmonic force, b) time-domain free oscillation. In a steady-state oscillation (b), the attenuation coefficient measures the absolute width of the resonance peak at the level of $1/\sqrt{2}$ of the maximum amplitude, and Q measures its relative width: $\Delta\omega_{1/2} = 2\chi = \omega_0/Q$. Example with $Q = 10$ shown.

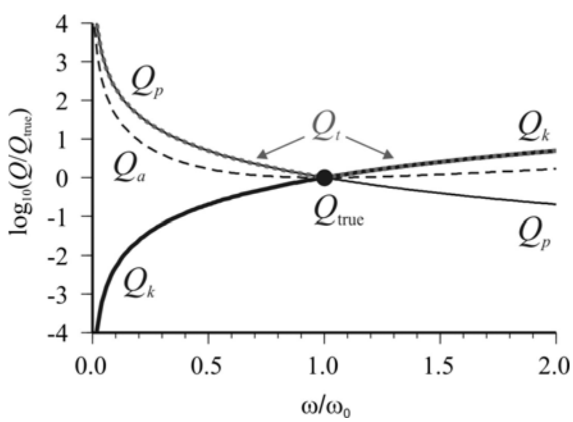

Figure 2. Frequency dependences of several types of Q definitions (16)–(19) for a linear driven oscillator. At the resonant frequency $\omega = \omega_0$, all of these values equal the true quality factor of the oscillator (12) denoted Q_{true} here.

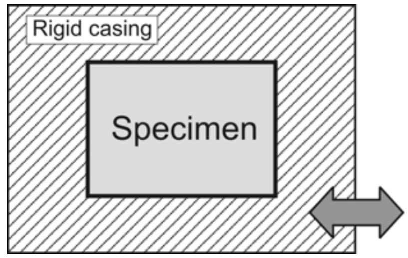

Figure 3. Thought experiment illustrating a problem with seismological *Q* definition (23). A rock specimen is welded into an infinitely rigid casing and subjected to an oscillatory motion without deformation.

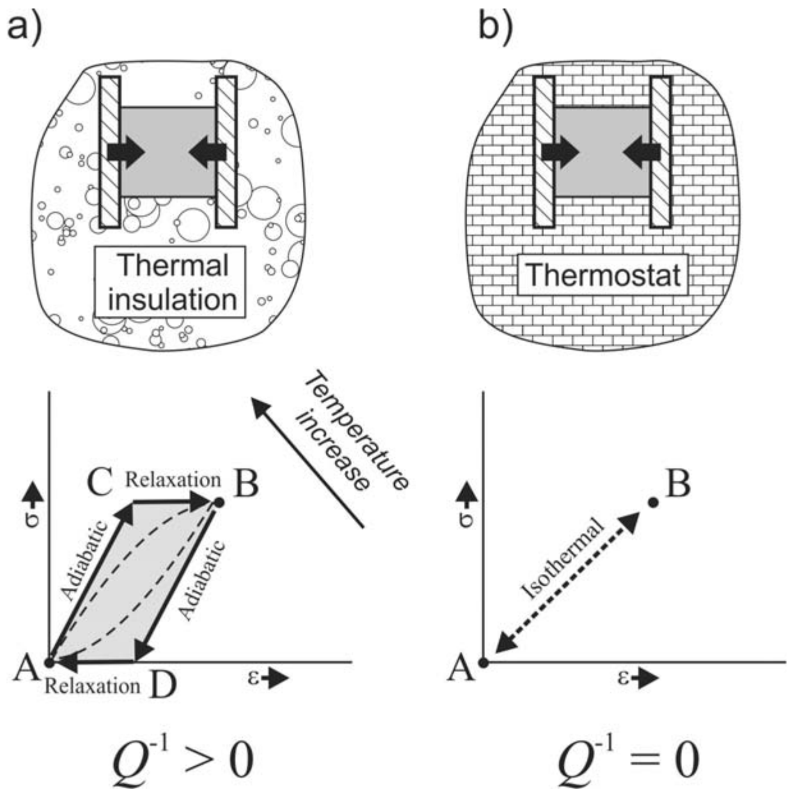

Figure 4. Thought experiment with cyclic deformation of a perfectly elastic material specimen (gray): a) adiabatic compression (AC) and decompression (BD) with thermal relaxation at constant stresses (CB and DA), b) isothermal compression/decompression in thermostat. Note that in case a), mechanical energy is dissipated (gray area of the hysteresis curve in the bottom plot).

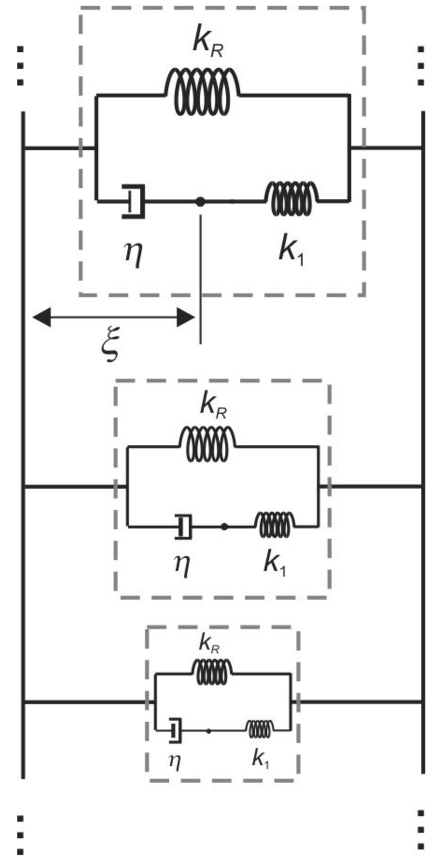

Figure 5. Mechanical model of a generalized viscoelastic solid [Liu *et al*. 1976]. Dashed boxes contain the standard linear solids (SLS) with different parameters, which are connected in parallel to implement the constitutive relation (26). In each SLS, k_R indicates the "relaxed" elastic modulus and η denotes the damping elements. Note the internal variables marked by dots and denoted ξ. The "unrelaxed" moduli are given by $k_U = k_R + k_1 - \eta\dot{\xi}(0)$, where $\dot{\xi}$ is taken at time $t \to 0$.

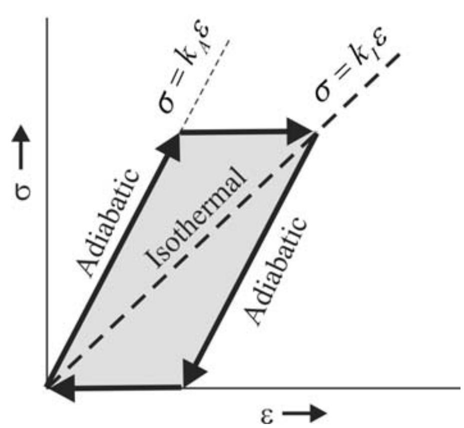

Figure 6. Differences between the adiabatic (k_A) and isothermal elastic moduli (k_I). The hysteresis loop from Figure (4a) is also shown for comparison (arrows).