Seismic attenuation without Q – I. Concept and model for mantle Love waves

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Summary

The attenuation quality parameter (Q) is a phenomenological quantity depending on the observations and on the underlying theoretical models. Q is a valid attribute of attenuating wave processes; however, as a property of the propagating medium, it is non-unique and can be completely fictitious. In many existing models, medium-Q factors are defined as properties of time-dependent viscoelastic moduli, and inversions are based on analytical extrapolations of the elastic parameters into the corresponding complex planes. Although these theories are mathematically self-contained, the resulting Q^{-1} values may lead to inaccurate or even unreasonable solutions when interpreted intuitively, as measures of energy dissipation. Overall, the attenuation coefficient α represents a significantly more robust representation of the in situ attenuation than Q. As an example, we show that the well-known viscoelastic solution for the long-period Love-wave Q_L within the Earth's mantle violates the conservation of energy and overestimates the attenuation levels. A new derivation is given, based on an explicit interpretation of the attenuation coefficient. The new Q_L in the Gutenberg's continental structure is 10-20% higher than before but shows a similar frequency dependence. Mantle attenuation

sensitivity kernels are different from the velocity sensitivity kernels. Attenuation kernels are sensitive to the largest particle velocities whereas phase velocity kernels – to the zones of highest strain within the subsurface. These differences should have significant implications for 1D and 3D modeling and inversions for the attenuation structure of the Earth.

1 Introduction

Seismic attenuation is typically observed by indirect methods, such as by using temporal amplitude decays of standing waves, widths of spectral peaks, or spatial decays of amplitudes in propagating waves at different frequencies. Once the frequency dependence of the quality factor, $Q(f)$, and the differences between elastic and anelastic attenuation have become recognized, many inversion methods were proposed to address these properties. Unfortunately, with typical data paucity and scatter, differences between wave types and datasets, and the increasing complexity of inversion techniques, attenuation models have become prone of significant uncertainties. It also appears that the basis of the Q concept and its influence on the character of seismological measurements have received insufficient attention (Morozov, 2009a, b).

Although this may seem surprising with the routine use of Q in today's seismological studies, the very existence of a quality factor describing the energy dissipation within the Earth is neither unequivocally obvious nor follows from solid-state or fluid mechanics. For propagating waves, the parameter that truly exists and is directly measurable is the spatial attenuation coefficient (*e.g.*, Chernov, 1960)

$$\alpha(f) = -\frac{1}{2}\frac{d\ln\left[E(r,f)\middle/G(r,f)\right]}{dr}. \quad (1)$$

In this expression, E is the wave energy density, r is the travel distance, f is the frequency, and $G(r,f)$ is the purely geometrical amplitude spreading that is assumed to be removed from α. In general, this attenuation coefficient is frequency-dependent, and by denoting its limit at $f = 0$ by γ^*, one can write (Morozov, 2008, hereafter M08):

$$\alpha(f) = \gamma^* + \frac{\pi}{Q_e^*(f)} f. \quad (2)$$

Thus-defined "effective" Q_e^* is analogous to the quality factor of a mechanical or electrical oscillator, in the sense of its being a dimensionless parameter representing the part of energy dissipation that increases with increasing frequency. Note that for a traveling wave (Figure 1), Q_e still measures the *local* dissipated energy density. However, the conventional Q is defined differently, and is based on the amplitude drop *cumulative in one wavelength* λ (Aki and Richards, 2002, p. 162):

$$Q(f) = \frac{\pi}{\alpha(f)\lambda} = \frac{\pi f}{\alpha(f)V}, \quad (3)$$

where V is the incident wave velocity. The important difference of eq. (3) from (1) is that a characteristic of the incident wave (wavelength λ) is embedded in it. This makes Q not entirely medium-controlled, and automatically leads to its proportionality to λ^{-1} or f for longer waves (Figure 1).

It is important to clearly differentiate between the properties of the elastic media and those of waves propagating it them, yet such differentiation is often obscure when using Q. Attenuation factors Q^{-1} are usually intuitively attributed to the media but justified only by considering plane- or spherical-wave solutions (*e.g.*, Aki and Richards, 2002). Such solutions only exist in uniform media, in which assigning the resulting Q^{-1} to the medium is trivial. In such cases, $\alpha(f)$ is nearly proportional to f, which compensates the artificial factor λ^{-1} in eq. (3). However, this proportionality only holds for such simple cases of perfectly-known geometrical spreading and disappears, for example, for surface waves. Quantities (3) and (1) would also have been equally applicable to the medium if there were reasons to assume that $\alpha \propto V^{-1}$; however, no such reasons exist. Thus, unlike $\alpha(f)$, the quality factor (3) is a phenomenological attribute of the specific wave rather than an *in situ* medium property.

Although the transformation of $\alpha(f)$ into $Q(f)$ (eq. 3) may seem natural for wave-like processes, its underlying assumptions are neither trivial nor entirely innocent. The very use of Q instead of the more general α eliminates the possibility of the zero-frequency attenuation coefficient $\gamma^* \neq 0$ in eq. (2). As shown in detail in Part II, non-zero γ^* results from variations of geometrical spreading, and values $\gamma^* > 0$ are commonly observed in short-period seismological data (M08). In the $Q(f)$ picture, such γ^* values lead to spurious frequency-dependent Q.

The above observation is just one illustration of the uncertainty of the notion of the "medium Q." In interpreting the Q values arising from various forward and inverse Earth models, it is important to keep in mind what type of quantity is being measured. In their Chapter 3, Bourbié *et al*. (1987) summarized a number of such measurements and noted that although most of them can be successfully described by the corresponding viscoelastic models, there is little agreement between the resulting values of *Q*. *Q* is simply unlikely to represent a common medium property.

As most researchers would agree, the general purpose of using a "medium *Q*" is for describing the ability of a unit volume of the medium to dissipate seismic energy. This is how the interpreters usually understand Q^{-1} within the Earth, associating its increased values with elevated temperatures, presence of fluids, or scattering. However, for theorists, the *in situ* Q^{-1} is often a different quantity. Because no single parameter describing the ability of the medium to dissipate the elastic energy exists, various phenomenological proxies were proposed (Bourbié *et al*., 1987). In particular, in formal visco-elastodynamics, a Q^{-1} is indeed *defined* as a local property of the medium, equal to the argument of the complex elastic modulus in the frequency domain (Anderson and Archambeau, 1964). Such Q^{-1} leads to detailed mathematical models of wave propagation and energy dissipation (*e.g*., Borcherdt, 2009). However, this still does not mean that complex elastic moduli are indeed present within the Earth, and that such Q^{-1} is related to any geologically-meaningful properties.

As shown in Sections 2 and 3, the concept of seismological *Q* is based on four mathematical conjectures, analogies, and assumptions. However, despite their elegance, long history, and tremendous impact on attenuation studies, all of these assumptions appear to fail. The existing *Q* is only a phenomenological quantity derived by extrapolating the attributes of simple wave solutions instead of analyzing the mechanical properties of the propagating medium itself.

In Section 4, in order to understand the process of attenuation from first principles, we outline an approach based on the Lagrangian wave mechanics. This description is well-known and commonly used in theoretical physics, and it was used in exploration geophysics (*e.g*., Biot, 1956; Bourbié et al., 1987). Nevertheless, this approach appears to be under-utilized in global seismology. This illustration shows that: 1) attenuation should not be physically connected to the complex-valued moduli, and 2) unfortunately, attenuation represents a significantly more complex phenomenon that cannot be treated by heuristic conjectures and assumptions. The general conclusion from this section is that again, attenuation parameters should be studied specifically and not mixed into the elastic properties.

Further in Section 4, we take an "interpretive" view on attenuation above and propose a model of seismic attenuation based on the notion of the attenuation coefficient distributed within the Earth. The "quality factor" *Q* and its associated viscoelastic mechanics is not included in this model, and its use should generally be discouraged. However, because of its overwhelming use in seismological literature, we still render the attenuation-coefficient expressions in terms of *Q* where it is possible without confusion. Further theoretical analysis of the attenuation coefficient and its relation to bending rays and reflectivity is presented in Part II of this study.

To illustrate the attenuation-coefficient approach, in Sections 5– 7, we revisit the study by Anderson *et al*. (1965) and perform modeling of the frequency-dependent (apparent) surface-wave *Q*. These classic results are selected for their simplicity and also because they laid the methodological basis for today's interpretations of *Q* and for numerous modern inversions. These analytical and numerical predictions of the Rayleigh and Love-wave *Q* have hitherto been unquestioned (Aki and Richards, 2002, p 290-291); nevertheless, our example shows that the viscoelastic approach violates the conservation of energy and results in ~10–20% over-estimated levels of attenuation. To derive a correct solution, we use the wavenumber-eigenvalue approach (Aki and Richards, 2002) which can also be related to the frequency- eigenvalue, spherical harmonic summation used in whole-Earth studies (Dahlen and Tromp, 1998; Gung and Romanovicz, 2004).

The clarity of the 1D problem allows us to focus on the underlying paradigms, conjectures, and theoretical assumptions which are imbued in numerous recent investigations of the Earth's 3D attenuation structure (*e.g.*, Gung and Romanovicz, 2004, and many other studies). However, in 3D tomographic inversion, understanding of the fundamental principles of attenuation modeling is complicated by many analytical and computational details. For example, Lekić et al. (2009) recently made a spectacular conclusion that merely by manipulating the sensitivity kernels, the "physical" frequency dependence of mantle Q^{-1} can be established beyond any doubt and without constructing a correct attenuation model. Nevertheless, the analysis presented below questions the physical significance, accuracy, and potentially even the existence of such kernels for Q^{-1}. Because of the forward modeling failing in the 1D case, it is obvious that any inversion and subtle interpretations, and particularly in 3D, should also become problematic.

2 Basis of the "medium-Q" concept

The key theoretical observation facilitating both the early 1D (Anderson et al., 1965) and modern 3D attenuation inversions (Gung and Romanovicz, 2004) is that the Fréchet sensitivity kernels K_q relating the *in situ* surface-wave properties q of to the observed ones, q_{obs}:

$$q_{obs} = \int K_q(\mathbf{r}_{obs}, \mathbf{r}) q(\mathbf{r}) d^3\mathbf{r} \,, \tag{4}$$

are the same for q taken equal to VQ^{-1} or to the wave velocity V within the medium. This statement is closely related to the interpretation of Q^{-1} as a negative complex argument of the medium velocity (eq. 5.93 in Aki and Richards, 2002). Physically, this equivalence is most amazing and worrisome, considering that many factors control the energy dissipation within the Earth, such as fracturing, fluid content and saturation, viscosity, porosity, permeability, tortuosity, properties of "dry" friction on grain boundaries and faults, and distributions of scatterers (Bourbié *et al*., 1987). Most of these factors are only remotely (at best) related to the velocity. The ability to lump them all together in a cumulative medium Q^{-1} suggests that only some specific wave mode is in fact considered, and its properties are substituted for the properties of the medium.

Indeed, Q^{-1} in eq. (3) is not the type of quantity that can be uniquely attributed to any point in the medium, as parameter $q(\mathbf{r})$ in eq. (4). Values of Q^{-1} are different for different waves (for example, P, S, and various inhomogeneous waves – see Borcherdt, 2009). This difference is attributed to the two elastic parameters of the medium, such as the bulk and shear moduli (Anderson and Archambeau, 1964). However, let us ask ourselves, what properties of the *elastic moduli* λ and μ led to their association with attenuation? In answering this question, four fundamental hypotheses can be recognized in the pioneering studies of the Earth's attenuation in the 60's and 70's:

H1) In the frequency domain, complex-valued elastic moduli can be used to write the anelastic equations of motion. Conceptually, complex moduli arise from the popular interpretation of attenuation as "imperfect elasticity" (Anderson and Archambeau, 1964) presenting the stress (σ) as a convolutional response to the strain-rate history, $\dot{\varepsilon}(t)$ (Dahlen and Tromp, 1998):

$$\sigma(t) = \int_{-\infty}^{t} M(t - t')\dot{\varepsilon}(t')dt' , \quad (5)$$

where $M(t)$ is the generalized time-dependent viscoelastic modulus. This interpretation is closely related to the relaxation-spectra (*e.g.*, Liu et al., 1976) and equivalent mechanical models, and also to the correspondence principle (*e.g.*, Bland, 1960). It was supported by extrapolating the results of creep measurements (Lomnitz, 1956) to the seismic frequencies; however, this was also done indirectly, with the use of the relaxation law (5) and equivalent models.

H2) Phase-velocity dependencies on the medium parameters can be analytically extrapolated into the complex plane in order to derive the attenuation properties (*e.g.*, Anderson and Archambeau, 1964). This conjecture directly led to the similarity of the forward velocity and Q^{-1} kernels in eq. (4).

H3) The Q parameter is expected to vary with frequency, and the power-law frequency dependence $Q(f) = Q_0 f^{\eta}$ is often suitable for describing the observed (apparent) and also material attenuation (*e.g.*, Aki and Chouet, 1975; Anderson and Given, 1982).

H4) Geometrical spreading can be "reasonably" accurately modeled, which allows correcting the observed amplitudes for it and measuring the frequency-dependent Q.

These hypotheses form the basis of both frequency-dependent attenuation modeling and measurements and are rarely questioned today. Viscoelastic theories based on these assumptions are elegant, mathematically rich, and self-consistent (Carcione, 2007; Borcherdt, 2009).

Unfortunately, all four assumptions (H1-H4) above appear to be inadequate. They only describe the reality in a very limited number of cases, and the case of a surface wave in a layered Earth is not among them. Assumption (H4) may be the most harmful, because it affects the very procedure of data measurement and presentation (Morozov, 2009a). As recently demonstrated by revisiting several key studies (M08, Morozov, 2010a,b) geometrical-spreading models are insufficiently accurate in most observational cases, and their corrections often eliminate the need for a frequency-dependent *in situ* Q. Assumption (H3) can be viewed as only a convenient parameterization for Q; however, in conjunction with (H4), it leads to incorrect values and spurious frequency dependences of Q in cases of imperfectly-corrected geometrical spreading (Morozov, 2009a,b).

In this paper, we focus on the role of the fundamental assumption (H1), *i.e.* of the constitutive law for attenuation. The correspondence principle unquestionably describes real wave propagation only in a homogenous medium, because only in such medium there exists an elastic parameter directly corresponding to the phase velocity (such as $V_S = \sqrt{2\mu/\rho}$ for S waves, where μ is the rigidity modulus and ρ is the density). This allows attributing the imaginary shift in the phase velocity to μ, thereby justifying the relaxation model (H1) above. However, in a heterogeneous, attenuative medium, the phase velocity does not equal the material velocity at any point, and therefore the reasons for introducing a complex-valued material μ remain unclear. As shown below, even assuming that all attenuation factors can be summarized in a single material Q^{-1}, they should still not be attributed to the elastic moduli.

3 Types of Q models

The Q factor is a phenomenological quantity dependent on the type of observation conducted with the deformation-stress field. Quasi-static experiments, such as creep and relaxation (*e.g*., Lomnitz, 1956) first lead to the convolutional laws as in eq. (5), and in some cases, such laws could be implemented in rheological models (*e.g*., Carcione, 2007, sections 2.4–5). However, within the seismic frequency bands, creep and relaxation are still inferred only implicitly through the interpreted quality factor Q. In different experimental environments (wave propagation, forced harmonic oscillations, or free vibrations), different values of Q arise from the relaxation models, and they cannot be reduced to each other (for an overview, see Chapter 3 in Bourbié et al., 1987). For example, the Biot theory for porous saturated rock, or scattering theory (*e.g*., Chernov, 1960) clearly lead to the spatial attenuation factors α (eq. 1). These factors can be converted into Q by using eq. (3); however, such Q's are different from those determined in the experiments with resonant bars (White, 1983). Thus, the generalized viscoelastic model (5) can satisfactorily represent and compare the various field problems, yet its significance in relation to the fundamental medium properties should not be overstated. These models can be viewed as working mathematical tools rather than an end in themselves (Bourbié *et al*., 1987).

Formulations of the dynamics of deformation in the literature can generally be subdivided into two groups. First, in the “axiomatic” approach usually referred to as the linear viscoelasticity (Bland, 1960), a rigorous mathematical theory is constructed by starting from the constitutive law (5). This approach is broadly used in theoretical global seismology (Dahlen and Tromp, 1998). Unlike rigorous theoretical mechanics, this theory does not use the Hamilton variational principle but starts from differential equations of motion and relies on dashpot-spring analogies for their support (*e.g.*, Carcione, 2007). Further description is entirely self-consistent and close to that of an elastic problem, from which it differs by using complex-valued elastic constants in the frequency domain. Nevertheless, this difference also leads to new types of solutions (such as inhomogeneous and multiple *S* waves), which sometimes possess peculiar properties (Richards, 1984).

In the frequency-domain form of the linear viscoelastic model (eq. 5), the attenuation is described by negative phase shifts of the medium velocities V_P and V_S, (*e.g.*, Aki and Richards, 2002), and Q^{-1} is defined as a phase shift between the complex-valued particle velocity and stress. This phase shift is further attributed to the arguments of the complex elastic moduli, whereas the possibility of an imaginary component of density is not considered (Anderson and Archambeau, 1964; Borcherdt, 2009). However, such extrapolation of Boltzmann's (1874) after-effect theory (which describes relaxation phenomena such as creep) to seismic frequencies is still phenomenological and appears to disregard several key mechanical principles. Firstly, if the attenuation is caused by friction on grain boundaries and faults, or by viscosity in pore fluids, it should affect the kinetic, and not the elastic-deformation energy. Strain energy dissipates into heat not by means of some "imperfect elasticity" but by causing relative movements within the medium. The Lamé parameters λ and μ describe the elastic energy stored in the field, and they should not be responsible for attenuation. Secondly, attenuation is a property of a particular wave and corresponds to an imaginary part of its wavenumber or frequency. The key idea of visco-elastodynamics can be summarized as attributing an attenuation property (namely, Q^{-1}) of a plane *S*-wave in a uniform medium to the local shear modulus, and that of the *P*-wave – to the local modulus *M*. This mathematical conjecture provides a great short-cut in the theory; however, it is not necessitated by the physics. Its predictions start deviating from reality when considering inhomogeneous media and boundary conditions, which also become "anelastic" in this approach (Borcherdt, 2009). For example, when compared to the derivation based on the traditional wave equations, visco-elastodynamics gives opposite signs of the phase shifts for reflections from attenuation contrasts (Lines et al., 2008), which was explained by incorrect expressions for the anelastic acoustic impedance (Morozov, 2010c).

By contrast, the approach that can be called "physical" attempts building a wave-propagation model by using the traditional mechanics, which describes the energy dissipation by viscous flows or dry friction. Apparently because of its attention to fluids, this approach is more developed in exploration seismology (*e.g.*, Bourbié et al., 1987). The medium is described by using the Lagrangian formulation, in which the dissipation is considered separately from elasticity, and all medium parameters remain real. Notably, when certain solutions to the wave equations are considered (for example, harmonic plane waves in homogeneous media), phenomeonological complex moduli may also arise (Bourbié *et al*., 1987; Carcione, 2007).

The second of these approaches is far more preferable for unraveling the true physics of wave propagation, and Lagrange formulation is well-known for its depth, power, and generality. To illustrate this model of elastic energy dissipation, we only summarize its principles below; however, their relation to the full wave problem (and also its complexity in real-world applications) are quite apparent. For more complete treatments of viscoelasticity in realistic porous media, see Bourbié *et al*. (1987) and Carcione (2007).

4 Attenuation in Lagrangian elastic-medium mechanics

In Lagrangian form, the dynamics of any mechanical system (such as the elastic field) is described by a function of some generalized coordinates $\mathbf{q}$ and their time derivatives $\dot{\mathbf{q}}$

$$L(\mathbf{q},\dot{\mathbf{q}}) = E_{kin} - E_{el}\,, \tag{6}$$

where E_{kin} and E_{el} are the kinetic and elastic energies. Vector $\mathbf{q}$ consists of any parameters describing the field (*e.g.*, local displacements, their Fourier amplitudes, or Rayleigh-Ritz coefficients below), and $\mathbf{q}$ and $\dot{\mathbf{q}}$ are treated as independent variables. The corresponding Euler-Lagrange equations of motion are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0\,,\ \ i = 1,2,\ldots \tag{7}$$

In the presence of energy dissipation, these equations are modified by adding the generalized dissipative force Q_i^D (do not confuse with the quality parameter Q):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i^D\,, \tag{8}$$

which is a derivative of some "dissipation function" ("pseudo-potential") D in respect to $\dot{\mathbf{q}}$:

$$Q_i^D = -\frac{\partial D}{\partial \dot{q}_i}\,. \tag{9}$$

For a simple mechanical analogy, consider a linear oscillator of mass *m*. Its Lagrangian is

$$L(\mathbf{r},\dot{\mathbf{r}}) = \frac{m}{2}\dot{\mathbf{r}}^2 - \frac{m\omega_0^2}{2}\mathbf{r}^2 . \tag{10}$$

where **r** is the vector of its Cartesian coordinates, and ω_0 is its natural frequency. If the force of viscous friction is linear in velocity:

$$\mathbf{f}_D = -\xi m\omega_0 \dot{\mathbf{r}} , \tag{11}$$

then the corresponding Rayleigh dissipation function is (*e.g.*, Razavy, 2005)

$$D = \xi \frac{m\omega_0}{2}\dot{\mathbf{r}}^2 . \tag{12}$$

If the oscillator is driven by an external force with frequency ω, the energy δE dissipated in one period $T = 2\pi/\omega$ is

$$\delta E = \int_0^T \xi m\omega_0 \dot{\mathbf{r}}^2 dt = \xi\omega_0 T E_{kin} . \tag{13}$$

where E_{kin} is the peak kinetic energy in that period.

For a mechanical oscillator, the quality factor is simply a constant parameter of the system, denoted $Q = \xi^{1}$. Consequently, there is no question of its frequency dependence. However, if the quality factor is defined “as in seismology” (Aki and Richards, 2002, p.162), then it becomes linearly increasing with frequency:

$$Q = \frac{2\pi E_{kin}}{\delta E} = \frac{\omega}{\omega_0 \xi} , \tag{14}$$

because δE increases with period (eq. 13). This expression explains one fundamental reason for observing the seismological Q often quickly increasing with frequency: such frequency dependence is simply encoded in its definition. It is "natural" to expect $Q \propto \omega$ in systems with frequency-independent energy dissipation.

Further, taking into account eq. (12), the equation of motion (8) with viscous friction becomes

$$m\ddot{\mathbf{r}} = -m\omega_0^2\mathbf{r} - \xi m\omega_0\dot{\mathbf{r}}\,, \tag{15}$$

and its general solution is $\mathbf{r}(t) = \mathrm{Re}[\mathbf{A}\exp(-i\omega'_0 t)]$, where

$$\omega'_0 \approx \omega_0\left(1 - \frac{i\xi}{2}\right) \tag{16}$$

is the "complex frequency," and **A** is an arbitrary complex-valued amplitude. A similar complex frequency arises when describing the attenuation of Earth's normal modes (Dahlen and Tromp, 1998, Chapter 6). Following the "viscoelastic" practice, one can therefore define a complex-valued spring constant (an equivalent to the elastic modulus)

$$k' = m\omega'^2_0 \approx k(1 - i\xi), \tag{17}$$

and eq. (15) then takes the form of the free-oscillator equation (12):

$$m\ddot{\mathbf{r}} = -m\omega_0'^2\mathbf{r}\,. \tag{18}$$

If an external force $f(t)$ is considered in the right-hand side of eq. (15), then its frequency-domain solution can also be expressed by using the complex frequency alone:

$$A(\omega) = \frac{f(\omega)}{m\left(\omega_0'^2 - \omega^2\right)}. \tag{19}$$

This formula resembles a single-mechanism "relaxation spectrum" (Liu *et al.*, 1976).

Thus, expressions (6–14) give a consistent analog to the elastic-wave attenuation processes. Eqs. (16–19) show how the complex-valued elastic moduli and relaxation spectra arise, but only because of the existence of an oscillatory solution with $\omega = \omega_0$. The fundamental difference of the propagating wavefield case is that it has no "natural" oscillation frequency arising from the wave equation. Without special relaxation mechanisms, the medium is non-resonant, and the oscillations are of a purely forced nature. The energy dissipation occurs per unit volume or travel path and can be measured by the attenuation coefficient, α (Chernov, 1960; Bourbié *et al.*, 1987). In order to obtain a Q value, α has to be arbitrary converted to Q by using eq. (3) and introducing a frequency- or wavelength dependence similar to the one shown in eq. (14).

Eq. (8) shows that the attenuation is caused by external friction forces which *are not included* in the Lagrangian (*i.e.*, in μ or λ). The effect of friction can nevertheless be described by an energy-like function D (eq. 12), but the function is similar to the kinetic, and not to the elastic energy. For example, in a porous medium (Bourbié *et al.*, 1987, p. 69-72), the dissipation function is quadratic in filtration velocity w_i (the velocity of the fluid relative to the rock matrix):

$$D = \frac{\eta}{2\kappa} \dot{w}_i^* \dot{w}_i \, , \tag{20}$$

where η is the fluid viscosity, and κ is the absolute permeability, which depends on the geometry of the pores.

The above shows that if taken literally, the association of friction with μ (which is a measure of shear-strain energy) is problematic. Instead of μ, the attenuation could also be related to an imaginary part of ρ, although this seems to be neither necessary nor productive as well. Anderson and Archambeau (1964) also discarded such possibility of Im$\rho \neq 0$, but because of its perceived association with "imperfect gravity" (which was also somewhat imprecise). However, note that that in porous saturated media, the density factor nevertheless does change in the presence of energy dissipation. Relative movement of pore fluids causes an additional inertial coupling force (Bourbié *et al*., 1987):

$$F_{inertial} = \rho_f (1 - a)\ddot{u}_i , \tag{21}$$

where $a \geq 1$ is the tortuosity parameter, and ρ_f is the pore fluid density. This force is proportional to the acceleration and effectively modifies the inertial property of the rock (*i.e*., its density ρ) to $\rho + \rho_f(1-a)$.

In summary of this section, it appears that phenomenological analogies and analytic extrapolations should be carefully scrutinized when used to infer wave attenuation within the Earth. The fundamental equation (8) and realistic multi-phase models based on the specific physics of dissipative processes should be much more reliable than such analogies.

5 Existing Love-wave Q(f) model in a layered Earth

The derivation of the surface-wave $Q(f)$ measured on the surface of a layered-Earth model by Anderson *et al*. (1965) (also see Aki and Richards, 2002, p. 289–291) was based on two of the heuristic conjectures above: (H1) interpretation of the attenuation parameter Q^{-1} as a phase shift of the complex-valued phase velocity:

$$\delta \ln c = -\frac{i}{2}\left(\text{spatial}\, Q^{-1}\right), \tag{22}$$

and (H2) assumed analytical dependence of the surface-wave phase velocities c on the velocities of the individual mantle layers ($V_{P,i}$ and $V_{S,i}$):

$$c = f\left(V_{P,i}, V_{S,i}, \rho_i\right), \tag{23}$$

where all properties also depended on frequency ω. It was also assumed that eq. (22) could be applied to both the apparent Q^{-1} observed on the surface and to the in situ Q (in combination with the medium velocity replacing c in eq. (22)). This allowed extrapolating the partial derivatives of phase velocities, such as $\partial c/\partial V_{P,i}$, into the corresponding complex planes, which further allowed calculating the surface-wave attenuation by using the phase-velocity derivatives. This approach yielded for Love waves (eq. 7.88 in Aki and Richards, 2002):

$$\text{spatial}\, Q_L^{-1} = \int_0^\infty K_V Q_S^{-1} dz\,, \tag{24}$$

where z is the depth, K_V is the S-wave velocity sensitivity kernel

$$K_V = \frac{\left[k^2 l_1^2 + \left(\frac{dl_1}{dz}\right)^2\right]\mu}{k^2 \int_0^\infty \mu l_1^2 dz}\,, \tag{25}$$

k is the wave number, μ is the Lamé rigidity modulus, Q_S is the shear-wave attenuation quality factor of the mantle, and l_1 is the amplitude of the mode of interest. All values in eqs. (24) and (25) are depth-dependent, so that the SH-wave displacement is given by

$$u_y(x,z,t) = l_1(z)\psi(x,t), \tag{26}$$

where $\psi(x,t) = \exp(-i\omega t + ikx)$.

Despite its simplicity, approach (22–23) nevertheless leads to significant difficulties. Its key problems are: 1) treating Q_S^{-1} as a fundamental medium property, which can also be considered local (such as μ) and 2) mixing the notions of wave speeds as parameters of the propagating medium and phase velocities of the various wave modes in it. Note that by its definition, the spatial Q for any wave mode corresponds to the complex argument of its wavenumber (*e.g.*, Aki and Richards, 2002, p. 167–169):

$$k \to k + i\alpha = k\left(1 + \frac{i}{2Q}\right), \tag{27}$$

where α is the spatial attenuation coefficient. Positive signs of Q ensure the amplitudes decaying in the directions of propagation. Through its relation to the phase velocity $c = \omega/k$, a positive Imk corresponds to negative Imc. However, in a surface wave, a single value of k is common to all depth levels, and velocities V_P and V_S in eq. (25) do not serve as phase velocities for any waves. Therefore, the inferred negative imaginary parts in V_P and V_S similar to that of the phase velocity (22) represent heuristic extrapolations of the dispersion property of the plane-wave solution ($c = \omega/k$) far away from the region of its validity.

Further, let us consider the hypothesis of analyticity (H2 above). Function (23) represents an integral transform of the layer parameters (see eq. 25), and its analyticity in respect to the functional integrands $V_{P,S}(z)$ (or equivalently, $\lambda(z)$ and $\mu(z)$) is unlikely in the general case. Thus, we need to avoid treating the phase velocity and attenuation as a single holomorphic function, and consider them independently.

Regarding the properties of the resulting solution, formula (25) is also problematic, because it violates the total energy balance by exaggerating the amount of energy dissipation. Values of Q_L^{-1} are presented as weighted averages of Q_S^{-1} within the layers; however, the weights in the numerator of this ratio are systematically greater than those in the denominator. This flaw of expression (25) can be easily seen on an example of a two-layer model with different velocities but constant in situ Q_S. In such a model, assuming that the energy in each layer $l = 1,2$ can be subdivided into non-dissipating ($E_{n,l}$) and dissipating ($E_{d,l}$) parts, $E_{d,l}$ should decrease after time t by the same factor for both layers: $E_{d,l} \propto (1-\lambda) = \exp(-\omega Q_S^{-1} t)$. Consequently, the total relative energy dissipation rate should not exceed λ:

$$\frac{\delta \tilde{E}}{\tilde{E}} = \frac{\lambda \left(E_{d,1} + E_{d,2} \right)}{E_{n,1} + E_{d,1} + E_{n,2} + E_{d,2}} \leq \lambda \, . \tag{28}$$

Therefore, the spatial attenuation factor of $Q_L^{-1} = Q_S^{-1}$ can be expected if all mechanical energy dissipates in this process, and $Q_L^{-1} < Q_S^{-1}$ if only a part of it dissipates (such as the kinetic energy above). However, the conventional formula (25) predicts $Q_L^{-1} > Q_S^{-1}$ on the surface, showing dissipation of greater energy than present in the field, which is unrealizable.

6 Love-wave Q(f) from energy-balance constraints

An alternate expression for the effective spatial attenuation can be derived directly from the energy balance considerations. Consider an *SH* surface-wave field of form (26) in a layered, isotropic, and lossy medium, in which the horizontal wavenumber contains a positive imaginary term (eq. 27). Its time-averaged kinetic energy density is (Aki and Richards, 2002, section 7.3)

$$\left\langle E_{kin} \right\rangle = \left\langle \frac{1}{2} \rho \dot{u}_i \dot{u}_i^* \right\rangle = \frac{uu^*}{4} \rho \omega^2 l_1^2 , \tag{29}$$

where complex conjugation (denoted by the asterisk) accounts for the complex-valued wavefield amplitudes in eq. (26). The corresponding average elastic energy density is

$$\left\langle E_{el} \right\rangle = \left\langle \frac{1}{2} \lambda \varepsilon_{kk} \varepsilon_{nn}^* + \mu \varepsilon_{ij} \varepsilon_{ij}^* \right\rangle = \frac{uu^*}{4} \left[\mu k k^* l_1^2 + \left(\frac{dl_1}{dz} \right)^2 \right], \tag{30}$$

and the total energy

$$\widetilde{E} = \int_0^\infty \langle E_{kin} + E_{el} \rangle dz = \frac{1}{2} \left(\omega^2 I_1 + kk^* I_2 + I_3 \right), \tag{31}$$

where the energy integrals are

$$I_1 = \frac{uu^*}{2} \int_0^\infty \rho l_1^2 dz , \quad I_2 = \frac{uu^*}{2} \int_0^\infty \mu l_1^2 dz , \text{ and } \ I_3 = \frac{uu^*}{2} \int_0^\infty \mu \left(\frac{dl_1}{dz} \right)^2 dz . \tag{32}$$

The total elastic energy contained in a normal mode equals its kinetic energy (Aki and Richards, 2002, p. 284)

$$\omega^2 I_1 = kk^* I_2 + I_3 . \tag{33}$$

As argued above, E_{kin} can be viewed as the source of energy dissipation. For weak attenuation, its loss is continuously replenished from the potential energy through eq. (33). For horizontal surface-wave propagation (*i.e.*, because of the common factor uu^* in eqs. (32)), both E_{kin} and E_{el} at any depth should thus decrease with travel distance x as $\exp(-2\alpha x)$. The total dissipation is a sum of energy losses at each depth:

$$-\frac{d\tilde{E}}{dx} = -\int_0^\infty \frac{d\left\langle E_{kin} \right\rangle}{dx} dz = \int_0^\infty 2\alpha_i \left(z \right) \left\langle E_{kin} \right\rangle dz , \tag{34}$$

where we define $\alpha_i(z)$ as the "intrinsic" spatial S-wave attenuation coefficient at depth z. In the absence of information about the specific mechanisms of attenuation, the spatial attenuation coefficient α_i still can be viewed as a substantive property responsible for S-wave energy dissipation. To see this point for longer waves, consider plane or surface waves with a fixed frequency ω and wavenumber k. Although such waves would generally not satisfy the equation of motion (*i.e.*, $\omega/k \neq V_S(z)$, and the waves may represent vibrations forced by the adjoining layers), energy dissipation from within different volumes should occur independently (Figure 1)

$$\frac{\delta E(L)}{E} = -2\alpha_i(z)L . \tag{35}$$

Therefore, the local $\alpha_i(z)$ should depend on frequency but be nearly independent of k. In Part II, we give further theoretical examples of frequency- and wavenumber-independent α_i.

The following paragraphs in blue were added as a suggestion, in response to Jeroen's question about D. In the paper, this should probably better go in a separate section.

In terms of the Lagrangian model in Section 4, the dissipation function corresponding to expression (34) is

$$D = \alpha_i V_S E_{kin} = \chi_i E_{kin} . \tag{35.a}$$

When viewed as a function of particle velocities (as in eq. 12), this function has the meaning of the energy density dissipated at a given point within the medium. Under the present approximation of locally-isotropic medium and long wavelengths, χ_i can be viewed as an independent parameter describing the tendency of the medium to dissipate the seismic-wave energy. Physically, this parameter measures the portion of the kinetic energy that activates certain types of internal movements within the rock matrix, which are further transformed into heat or scattered elastic-wave energy. The mechanisms of these movements should be numerous and sensitive to the structure of the rock, its composition, physical state, pressure, and temperature. Although these mechanisms are not well understood at present, relation (35.a) could likely be used as a viable heuristic approximation. In terms of quantity χ_i both forward and inverse attenuation models can be formulated in ways similar to those of the existing Q^{-1} models.

Because the dissipation function is given by a multiplication in the frequency-domain (eq. 35.a), it would have a convolutional form in the time domain. In particular, the part of χ_i linearly increasing with frequency, $\kappa_i\omega/2\pi$, leads to the time-domain dissipation described by particle velocity convolved with the Hilbert transform of the acceleration:

$$D=\int_{-\infty}^{\infty}d\omega\dot{u}^*(\omega)|\omega|\dot{u}(\omega)=i\int_{-\infty}^{\infty}d\omega\dot{u}^*(\omega)\theta(\omega)\ddot{u}(\omega)=\int_{-\infty}^{\infty}d\tau\dot{u}^*(t)\ddot{u}_H(t-\tau). \qquad (35.b)$$

Thus, similarly to the viscoelastic theory, the simplicity of the frequency-domain formula (35.a) is achieved by a time-retarded form of the mechanism of interaction. It still remains to be established how realistic such mechanisms may be; however, the observations of constant κ_i values (M08, M10a, b) suggest that such mechanisms could be close to reality. Alternately, if we look for perfectly instantaneous but frequency-dependent form of dissipation function, it would require interactions other than the second-order in u. Such interactions would intermix wave amplitudes at different frequencies. Under the same approximation, spatial gradients or wavenumbers do not influence the dissipation process, and therefore it remains strictly localized in space.

Published mantle attenuation models are usually presented in terms of frequency-dependent plane *S*-wave quality factors of the medium Q_S^{-1}, which need to be transformed into α_i by using

$$\alpha_i(z) = \frac{\omega Q_S^{-1}(z)}{2V_S(z)}. \tag{36}$$

For the temporal attenuation coefficient, a similar transformation gives

$$\chi_i(z) = \frac{\omega Q_S^{-1}(z)}{2}. \tag{37}$$

Similarly to (35.a), this form is independent of V_S and often more convenient in describing the observed data (M08).

From eqs. (34), the total attenuation coefficient observed on the surface becomes

$$\alpha_L = -\frac{d\tilde{E}}{\tilde{E}dx} = \int_0^\infty K_\alpha \alpha_i dz, \tag{38}$$

where the resulting α_i-sensitivity kernel is

$$K_\alpha = \frac{\langle E_{kin} \rangle}{\tilde{E}}. \tag{39}$$

Unlike formula (25), expression (38) preserves the sum of the total propagating and dissipated energies. To compare this expression to the conventional formula (24), we can define the "Q_S-sensitivity" kernels

$$K_Q = \frac{U_L}{V_S} K_\alpha, \tag{40}$$

where V_L is the Love-wave phase velocity. These kernels predict the observed Q_L^{-1} from Q_S^{-1} values assigned to the layers within the Earth:

$$\text{spatial } Q_L^{-1} = \int_0^\infty K_Q Q_S^{-1} dz \,. \tag{41}$$

Note that K_Q is functionally different from the velocity kernel K_V in eq. (25). Finally, the temporal attenuation coefficient χ can also be derived from α_L: $\chi_L = \alpha_L V_L$.

Note that the introduction of attenuation ($\alpha_i > 0$) also slightly shifts the phase and group velocity spectra. To see this, consider the variational principle for finding the dependence of $l_1(z)$ on the depth (Aki and Richards, 2002, p. 284)

$$\delta \int_0^\infty \langle L(\mathbf{u}, \dot{\mathbf{u}}) \rangle dz = \frac{1}{2}\left(\omega^2 \delta I_1 - kk^* \delta I_2 - \delta I_3\right) = 0 \,, \tag{42}$$

where $L(\mathbf{u}, \dot{\mathbf{u}})$ is the Lagrangian density of the elastic field , and I_1, I_2, and I_3 are the energy integrals defined in eqs. (32). For a fixed ω, the absolute value of the corresponding wavenumber $|k|$ is obtained by solving the eigenvalue problem of eqs. (33) and (42). However, integrals (32) only depend on α_i via a common factor uu^*, and therefore $|k|$ is independent of attenuation. Consequently, with non-zero attenuation, the real part of the wavenumber decreases as

$$\operatorname{Re} k = \sqrt{|k|^2 - \alpha_L^2} \approx |k| \left(1 - \frac{1}{8Q_L^2}\right), \tag{43}$$

which corresponds to a negligibly small phase-velocity ($c = \omega/k$) dispersion due to attenuation (Anderson *et al*., 1965). From the variational principle (eqs. 33 and 42), group velocity remains real and changes accordingly (cf. eq. 7.70 in Aki and Richards, 2002):

$$U = \frac{\delta\omega}{\delta \operatorname{Re} k} = \frac{\operatorname{Re} k}{\omega}\frac{I_2}{I_1}. \quad (44)$$

7 Numerical model of mantle Love-wave Q_L

The Rayleigh-Ritz method provides efficient numerical solutions to the eigenvalue equations (33) and (42) (Wiggins, 1976). By approximating the functional form of $l_1(z)$ in terms of some appropriately selected basis functions $\phi_i(z)$

$$l_1(z) = \sum_{i=1}^{N} m_i \varphi_i(z), \quad (45)$$

where coefficients m_i comprise a discrete model vector **m**, integral equations (33) are transformed into a matrix eigenvalue problem:

$$kk^* \mathbf{m} = \mathbf{A}_2^{-1}\left(\omega^2 \mathbf{A}_1 - \mathbf{A}_3\right)\mathbf{m}. \quad (46)$$

In this expression, the discrete energy matrices are

$$A_{1,ij} = \frac{uu^*}{2}\int_0^\infty \rho\phi_i\phi_j dz, \; A_{2,ij} = \frac{uu^*}{2}\int_0^\infty \mu\phi_i\phi_j dz, \text{ and } A_{3,ij} = \frac{uu^*}{2}\int_0^\infty \mu \frac{d\phi_i}{dz}\frac{d\phi_j}{dz} dz. \quad (47)$$

Earth-flattening corrections (Aki and Richards, 2002) can be incorporated in integrals (47) in order to account for the Earth's sphericity. By solving this eigenvalue problem, all possible values of $|k|$ and the corresponding eigenfunctions (45) are obtained, from which the attenuation spectra (38) can be calculated.

For example, in the Gutenberg continental Earth model (Table 1), 45 cubic polynomial basis functions by Wiggins (1976) give a convenient decomposition for $l_1(z)$

(Figure 2). With the shear-wave Q values from the attenuation model MM8 (Anderson *et al*., 1965; Table 1), expression (38) yields the apparent frequency-dependent Love-wave α_L and χ_L (Figure 3).

Notably, the modelled attenuation coefficients show characteristic linear dependences on frequency, including one covering the entire long-period frequency band $f < 0.02$ Hz modelled in Anderson *et al*. (1965) (Figure 3a). Such linear $\chi(f)$ were observed in many datasets (M08; Morozov, 2009a, 2010a,b), and they can generally be explained by the corresponding linearity of the intrinsic $\chi_i(f)$. The derivative $d\chi_L/df$ is nearly constant within the Love-wave data band and corresponds to effective $Q_e \approx 120$ (Figure 3b). As discussed in Part II, values Q_e are dominated by the lowest level of intrinsic Q within the structure and are higher or equal to that level. Therefore, such low level of Q_e should arise from the contribution of low-Q layers between 38–100 km in the model (Table 1). Thus, the relatively thin, low-Q sub-crustal mantle should dominate the observed Love-wave attenuation at long periods.

Figure 4 compares the attenuation-coefficient results to those from Q-factor modeling by Anderson *et al*. (1965). Note that our Q_L values are consistently higher than those from the presently used formula (25) (grey dashed line in Figure 4). This difference is significant (10–20%) and sensitive to the underlying velocity and Q_S distributions within the upper mantle. Such a large discrepancy should affect the 1D inversion for mantle Q values, and 3D inversions based on the equivalence of the Q^{-1}- and velocity-sensitivity kernels could similarly be error-prone.

8 Discussion

Although derived for an old and relatively simple 1D case, this study has significant implications for modern, 3D attenuation investigations. First, it shows that the classic 1D case is still far from being solved. The 10–20% overestimation of Q_L^{-1} in the benchmark solution suggest that similar errors should be present in many sophisticated 3D modeling schemes inheriting the same conceptual background. The noted violation of energy balance originates from the fundamental assumptions of the method that is also likely to affect the more recent studies.

Second, if the attenuation is no longer viewed as a complex-plane rotation (as in eq. 22) of the medium velocity, it may require special analysis similar to the one presented here. The attenuation sensitivity kernel becomes different from velocity sensitivity (Figure 5), showing that inversions for attenuation structure should no longer be similar to velocity tomography. For example, note that the observed 60-sec Love-wave Q_L^{-1} is most sensitive to the near-surface, where particle velocities (and therefore friction and pore/fault fluid flows) are the fastest (solid black line in Figure 5). This is different from the phase and group velocities, which are most sensitive to the depth near the base of the crust, where the elastic strain is the strongest (dashed black line in Figure 5). Although this difference appears natural, it contradicts the traditional assumption of attenuation responding to the same structures as the velocity (Aki and Richards, 2002; Gung and Romanovicz, 2004).

Recognition of the difference between the α_L- and velocity-kernels removes the need of the assumption of analyticity (holomorphism; H2 above). Holomorphism is a very strong constraint on a complex function, which requires that the partial derivatives of its imaginary part are related to those of the real part by the Cauchy-Riemann equations. Once such a property is assumed, the many degrees of freedom (V_S plus numerous attenuation parameters, or at least α) collapse to a single one (V_S), and the sensitivity kernels to Q_S^{-1} and V_S automatically attain the same shapes in both 1D (Anderson *et al*., 1965) and 3D (Gung and Romanovicz, 2004). Inversion for Q_S^{-1} or Imμ thus becomes closely related to velocity tomography. However, the observed phase velocity and attenuation is related to the corresponding *in situ* properties by integral transforms (*e.g*., eq. 23) whose holomorphism is hardly likely, and it also can hardly be assumed for convenience. For example, because of the existence of shadow zones and triplications, the dependence of the apparent head-wave travel-times and velocities on the *in situ* velocities can be discontinuous and non-differentiable. However, an abandonment of this assumption would also mean that the attenuation kernels used in many 1D and 3D inversions may need to be revised.

Third, if the attenuation is treated separately and medium parameters are real-valued, the complex-moduli visco-elastodynamics becomes significantly limited in its scope. Surface-wave attenuation discussed here is just one example where the viscoelastic approximation leads to problematic results. Another such example where this approach appears to fail is the problem of the anelastic acoustic impedance. In the presence of attenuation, the complex phase of impedance is positive (Morozov, 2009c, 2010c), which is opposite to the one predicted by the standard formula $Z = \rho V$ with a complex medium velocity (eq. 22). Thus, it appears that significant deviations from the axiomatic visco-elastodynamics are found in heterogeneous media.

Unfortunately, the distinction between the traditional models based on hypotheses (H1– H4) and the one presented here lies in the realm of physical methodology which rarely receive adequate attention. Their difference is in whether we explain the attenuation by time-retarded elasticity (5) in some type of an "equivalent model" or look for the actual mechanisms of energy dissipation. If only considering simple wave processes, equivalent models and relaxation spectra can always be derived from, for example, pore fluid properties (Bourbié *et al*., 1987; Carcione, 2007). However, the number of such tractable problems is very limited and basically reduces to homogenous-media cases.

Elasto-dynamics should not be formulated to only reproduce some particular form of wave solutions. The Lagrangian variational formulation is the most fundamental and well-established, not tailored to any types of equations, and allows solving a broad variety of physical problems. The solution for Love-wave attenuation in eq. (38) is also not a final and complete solution but more of an *ad hoc* approximation based on the requirement of energy balance, assumption of a local similarity to *S*-wave dissipation at any depth level, and independent attenuation mechanisms operating at different frequencies. A complete solution would require modeling of crust/mantle fluid/flow properties, porosity, permeability, fracturing, and other physical effects which are not well understood at present.

Finally, it appears that the attenuation-coefficient approach provides a consistent basis for describing the theory, measurements, modeling, and inverting for attenuation properties within the Earth. Further theoretical development of this approach, including derivations attenuation coefficients caused by variable wavefront curvatures and reflectivity, is performed in Part II of this study.

9 Conclusions

Interpretations of the Earth's attenuation models may be ambiguous and inaccurate because of the use of the quality factor (Q) to describe the attenuation properties of the medium, and also because of its axiomatic interpretation in visco-elastodynamics. Modeling of the Q structure within the Earth and its inversion are based on several strong theoretical assumptions such as analyticity and the similarities of the attenuation and velocity sensitivity kernels. These assumptions appear to be inaccurate or incorrect. As an example, the accepted expression for Love-wave Q_L observed on the surface of a layered mantle model (Anderson *et al*., 1965) is found to violate the conservation of total energy.

The attenuation-coefficient formulation and Lagrangian wave mechanics provide reliable theoretical descriptions of attenuation processes. Using this approach, a new solution for mantle Love-wave attenuation is proposed, which provides an explicit energy balance. The resulting Love-wave Q_L in the combined Gutenberg and MM8 models is 10–20% higher than in the conventional model and shows a similar apparent frequency dependence. Most importantly, the new attenuation sensitivity kernels differ from the corresponding velocity kernels. These results should have significant implications for 1D as well as for recent 3D inversions for the Earth's attenuation structure.

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Tables

Table 1. Gutenberg's layered continental structure model (Aki and Richards, 2002) with Q_S values from model MM8 (Anderson et al., 1965)

Layer number	Depth to bottom (km)	ρ(g/cm^3)	V_P (km/s)	V_S (km/s)	Q_S
1	19	2.74	6.14	3.55	450
2	38	3.00	6.58	3.80	450
3	50	3.32	8.20	4.65	60
4	60	3.34	8.17	4.62	60
5	70	3.35	8.14	4.57	80
6	80	3.36	8.10	4.51	100
7	90	3.37	8.07	4.46	100
8	100	3.38	8.02	4.41	100
9	125	3.39	7.93	4.37	150
10	150	3.41	7.85	4.35	150
11	175	3.43	7.89	4.36	150
12	200	3.46	7.98	4.38	150
13	225	3.48	8.10	4.42	150
14	250	3.50	8.21	4.46	150
15	300	3.53	8.38	4.54	150
16	350	3.58	8.62	4.68	150
17	400	3.62	8.87	4.85	180
18	450	3.69	9.15	5.04	180
19	500	3.82	9.45	5.21	250
20	600	4.01	9.88	5.45	450
21	700	4.21	10.30	5.76	500
22	800	4.40	10.71	6.03	600
23	900	4.56	11.10	6.23	800
24	1000	4.63	11.35	6.32	800

Figures

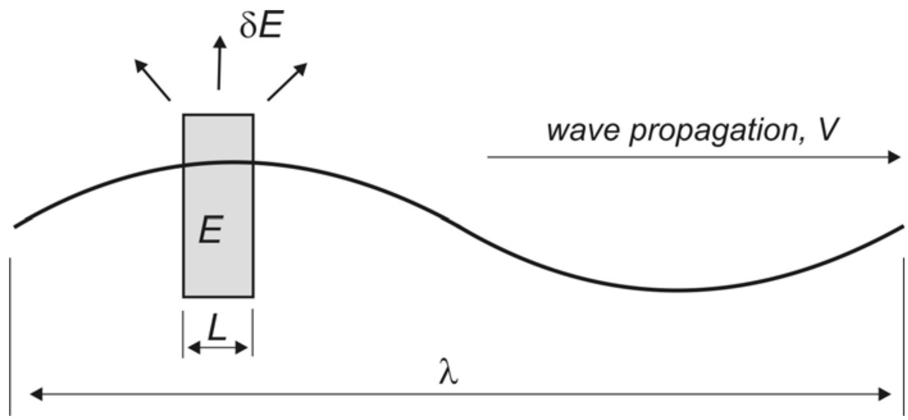

Figure 1. Energy dissipation problem (intrinsic or scattering) in eqs. (1-3) and (35). For typical sizes of dissipating volumes $L \ll \lambda$, the attenuation coefficient $\alpha = \delta E/E$ should depend on the frequency but be independent of λ.

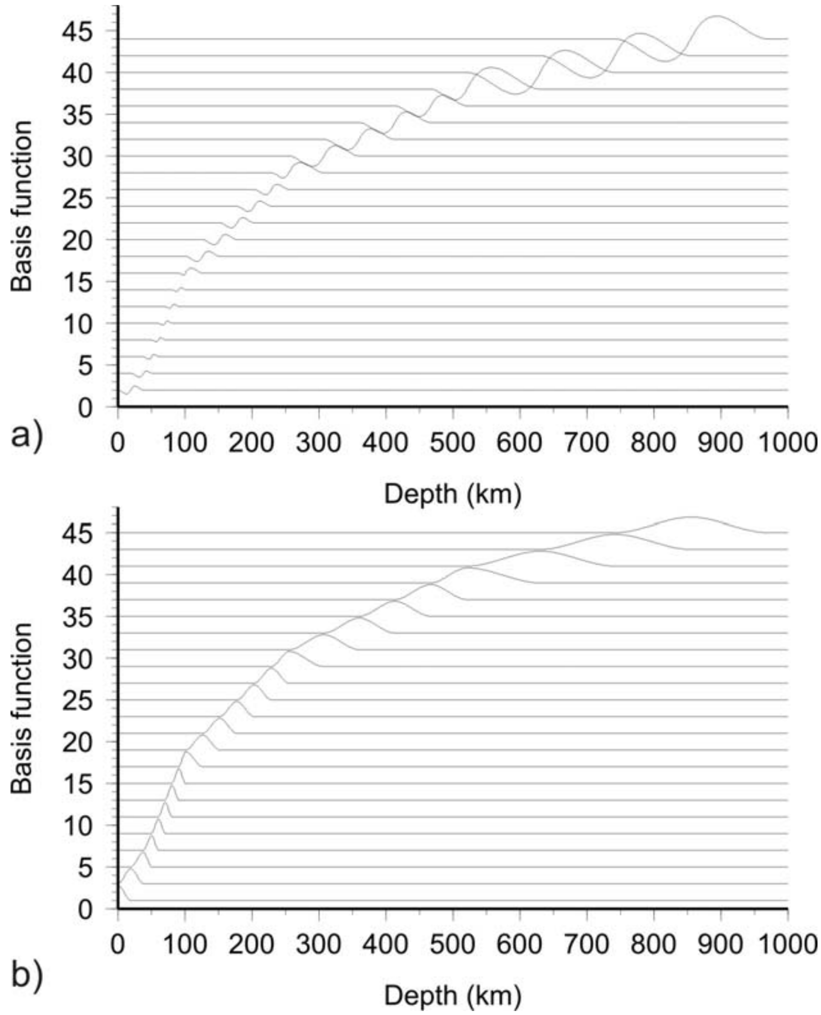

Figure 2. Basis functions used for modeling Love waves in Gutenberg Earth model (Table 1): a) functions normalized by $d\phi/dz = 1$ at layer boundaries; 2) functions normalized by $\phi = 1$ at the boundaries.

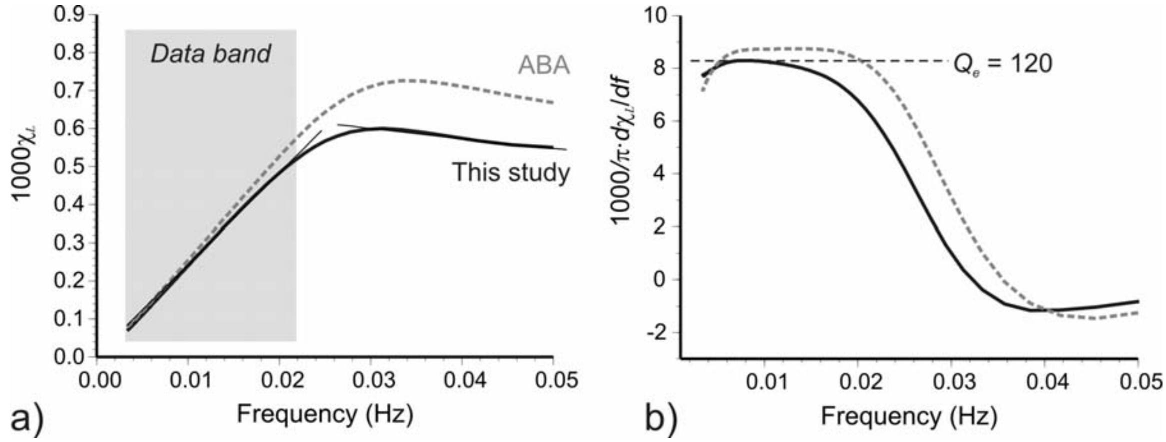

Figure 3. a) Attenuation coefficient $1000\chi_L$ in Gutenberg/MM8 model (Table 1) calculated by using expressions (38) (labelled "This study") and (25) ("ABA," after Anderson et al., 1965). Thin lines emphasize the linear trends. Grey box indicates the data frequency band inverted in Anderson et al. (1965). b) Derivative $1000/\pi \cdot d\chi_L/df$ emphasizing the two distinct levels of Q_e. Note the level $Q_e \approx 120$ within the data band.

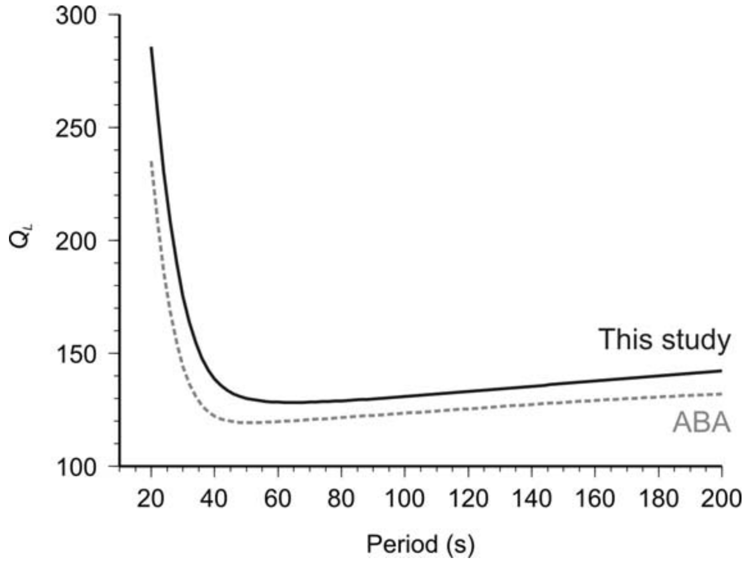

Figure 4. Apparent Love-wave Q_L predicted in the combined Gutenberg/MM8 model.

Labels as in Figure 3.

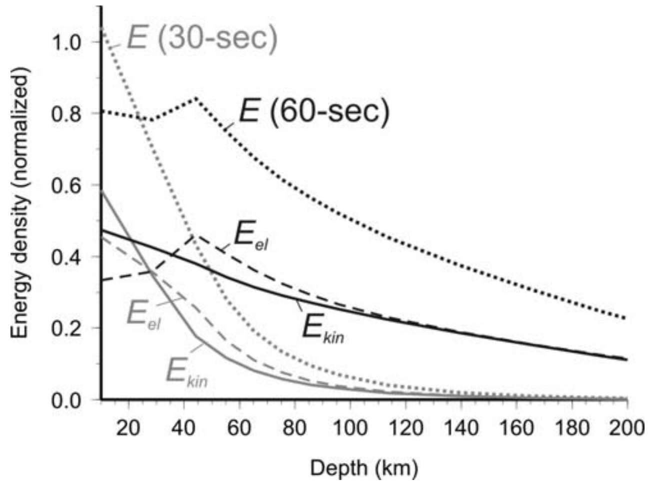

Figure 5. Normalized distributions of the kinetic (E_{kin}, solid lines), elastic (E_{el}, dashed lines) and total (E, dotted lines) energy density for the fundamental Love-wave modes at 60-sec (black) and 30-sec (gray) periods. Note that the E_{el} curve also represents the velocity sensitivity kernel K_V, and E_{kin} – Q_L^{-1} sensitivity K_Q in eq. (4).