Frequency dependence of long-period *t**

Igor B. Morozov

Department of Geological Sciences, University of Saskatchewan, Saskatoon, 114 Science Place, Saskatoon, SK S7N 5E2 Canada, Tel. 1-306-966-2761; Fax -1306-966-8593

Email*: Igor.Morozov@usask.ca*

Abstract

Multi-phase long-period t^* measurements are among the key evidences for the frequency- dependent mantle attenuation factor, Q. However, similarly to Q, poorly-constrained variations of Earth's structure may cause spurious frequency-dependent effects in the observed t^*. By using an attenuation-coefficient approach which incorporates measurements of geometric spreading (GS), such effects can be isolated and removed. The results show that the well-known increase of body P-wave t^* from ~0.2s at short periods to ~1–2 s at long periods may be caused by a small and positive bias in the underlying GS, which is measured by a dimensionless parameter $\gamma^* \approx 0.06$. Similarly to the nearly-constant t^* at teleseismic distances, this GS bias is practically range-independent and interpreted as caused by velocity heterogeneity within the crust and uppermost mantle. This bias is accumulated within a relatively thin upper part of the lithosphere and may be closely related to the crustal body-wave GS parameter $\gamma \sim$ 4–60 mHz reported earlier. After a correction for γ, P-wave t_P^* becomes equal ~0.18 s at all frequencies. By using conventional dispersion relations, this value also accounts for ~40% of the dispersion-related delay in long-period travel times. For inner-core attenuation, the attenuation coefficient shows a distinctly different increase with frequency, which is remarkably similar to that of fluid-saturated porous rock. As a general conclusion, after the GS is accounted for, no absorption-band type or frequency-dependent upper-mantle Q is required for explaining the available t^* and velocity dispersion observations. The meaning of this Q is also clarified as the frequency-dependent part of the attenuation coefficient. At the same time, physically-justified theories of elastic-wave attenuation within the Earth are still needed. These conclusions agree with recent re-interpretations of several surface, body, and coda-wave attenuation datasets within a broad range of frequencies.

Key words:

Attenuation; body waves; frequency dependence; geometric spreading; dispersion; inner core; mantle; structure; Q; t.*

1 Introduction

Attenuation of seismic waves within the Earth is among the most intriguing physical phenomena studied in seismology, and many attenuation models were developed in the past fifty years. With a number of uncertainties and differences between these models, a consensus about one fundamental point appears to be established by now, namely the frequency-dependence of the quality factor, Q. Across a broad range of frequencies from ~0.01 to 100 Hz (for summaries of several key datasets, see Jackson and Anderson, 1970; Anderson and Given, 1982; Doornbos, 1983; Lees *et al*., 1986; Abercrombie, 1998; Romanowicz and Mitchell, 2007), attenuation of surface and body waves consistently decreases, suggesting an increase in Q. This increase is often interpreted as an effect of the upper flank of the mantle absorption band, characterized by a relaxation constant $\tau_1 \approx 0.1–1$ s (Anderson *et al*., 1977; Doornbos, 1983). In lab measurements of seismic-wave attenuation, such an increase is also observed and often referred to as the high-frequency background (Cooper, 2002). The lower flank of the band is difficult to constrain and placed far below seismological frequencies (Anderson and Given, 1982).

Despite the growing number of models supporting the frequency-dependent Q within the crust and mantle, we noted recently (Morozov, 2008, 2009a, b, 2010a-c) that this question still remains open. Attenuation data are still limited, and interpretations can easily be biased, even involuntary, in favour of certain models. Most field observations of frequency-dependent Q share several fundamental weaknesses, which are:

1) Reliance on the viscoelastic theory rather than the traditional wave mechanics for describing seismic attenuation properties of the Earth;

2) Assumption that attenuation properties (for example, Q) depend specifically on the frequency rather than on the whole process of wave propagation;

3) Reliance on intricate models for Q (for example, frequency-dependent intrinsic and scattering Q's inferred from energy-flux equations) which are nevertheless built on overly simplified propagation models (such as $Amplitude \propto 1/Distance$). This produces a "model footprint" on the frequency dependence of Q, caused by the uncertainties and spatial variations of the Earth's structure. Models of microscopic dissipation mechanisms (such as movement of dislocations, diffusion, and kinetic equations) also leave a similar "footprint".

The first two points above are general, theoretical, and may be quite intricate. Starting from the early studies of energy dissipation in solids (Biot, 1956; Knopoff and MacDonald, 1958), it has been known that Q is difficult to rigorously express as a combination of physical properties of the material, and until now, the "medium Q" remains a heuristic attribute requiring scrutiny and explanation. These points were argued in Morozov (2009c, 2010c, d, 2011a, b, and in review) and will not be repeated here. The third point was also discussed on a number of data examples (Morozov, 2008, 2009a, b, 2010a, b; Xie and Fehler, 2009) and even led to a forum in *Pure and Applied Geophysics* (Mitchell, 2010; Xie, 2010). Our principal message there was that unfortunately, model assumptions leave a substantial "model footprint" even on the measurement of raw Q values, particularly when targeting a frequency-dependent Q or separating the intrinsic and scattering Q's. This model footprint needs to be examined specifically for each dataset, taking into account the nature of the waves involved and the uncertainties of the specific procedures of data measurement. In the present paper, we perform such analysis for long-period body waves.

Structural variability causes numerous complexities in the wave propagation process (such as bending rays, reflections or scattering for body waves), which in a first-order approximation can be described by the geometric spreading (GS). Although the sensitivity of Q measurements to GS is well known (*e.g.*, Kinoshita, 1994), its implications and extent are rarely appreciated in full. Major simplifications of the background Earth model are commonly made in order to infer a frequency-dependent Q. Only recently, by measuring the empirical GS, Morozov (2008, 2010a,b) showed that its variations can absorb *the entire* frequency dependences of Q reported in many cases of body, surface, and coda waves.

One key data type still not examined in the light of the variable GS is the multi-phase, and particularly long-period body-wave t^* and dispersion-related delay times. Such observations are considered in the present paper, in which we revisit several results by Der *et al*. (1982, 1986a,b) and Lees *et al*. (1986). In extensive studies, these authors, and also Der and McElfresh (1976, 1980), Der *et al*. (1985), Der and Lees (1985), and Sharrock *et al*. (1995) examined the dependencies of teleseismic t_P^* and t_S^* on the frequency and tectonic types of the lithosphere in several areas of the world. Broadly, these observations showed that: 1) for body waves beyond ~25° distance ranges, t^* values are nearly independent of the travel times (Der and Lees, 1985), 2) t^* values increase in tectonically-active areas, where a zone of increased Q^{-1} is present (Niazi, 1971; Sharrock *et al*., 1995), and 3) t_P^* decreases with frequency from ~1 s within the long-period band to ~0.2 s at short periods (Der and McElfresh, 1980; Der *et al*., 1982).

Until now, the above variations of teleseismic t^* were explained by the *in situ* Q^{-1} increasing within tectonic areas and decreasing with frequency, respectively. Nevertheless, similarly to Q, $t^* = \int Q^{-1} dt$ is an "apparent" quantity, and inferences about its relation to the *in situ* Q of the mantle (if such a quantity exists) should be done with caution. For body waves, the above model footprint is the effect of variable GS or scattering within lithospheric structures, which lead to spurious frequency-dependent t^* and Q. For example, by numeric coda modeling in realistic structures, Morozov *et al*. (2008) showed that increased GS rates (*i.e*., structurally more complex lithosphere) causes the apparent coda Q to increase with frequency.

As shown below, t^* measurements bear many features common with Q, including a strong trade-off with the GS. In frequency-dependent Q measurements, the discussions of uncertainties usually focus on the trade-off of Q with the assumed GS, which is usually taken in standardized forms, such as $t^{-\nu}$ with sometimes frequency-dependent exponents ν (*e.g*., Frankel *et al*., 1990; Zhu *et al*., 1991; Kinoshita, 1994; Yang *et al*., 2007). However, the true difficulty of interpreting $Q(f)$ for the physical properties of the subsurface is not in selecting an appropriate form for the GS law but in accounting for the unknown variations of GS within the Earth and for its dependence on wave types. With *any* reference GS model selected, variable GS would cause frequency-dependent artefacts in the resulting Q or t^*.

For reliable interpretation of the *in situ* attenuation, it should be described in a manner invariant in respect to both the unknown GS variations and mathematical models. Such a description can be achieved by using the temporal attenuation coefficient (denoted χ here) in both measurements and modeling. Below, we apply this method to t^* and Q^{-1} data. These two cases are discussed in parallel, with emphasis on the effects of GS variability.

In addition to teleseismic P waves, t^* and the associated velocity dispersion provides important information about the attenuation within the inner core (Doornbos, 1983; Bhattacharyya *et al*. 1993; Cormier and Li, 2002; Li and Cormier, 2002). This case is also briefly considered here by using the key observations by Doornbos (1983). Compared to the upper-mantle body waves, a distinctly different frequency-dependent t^* is observed for the inner core, in which we note a remarkable similarity of $t^*(f)$ dependences to those predicted in saturated porous rock (Biot, 1956). For the inner core, we similarly argue that the absorption band may be unnecessary.

Before starting the analysis, a disclaimer regarding the meaning of Q and its frequency dependence seems appropriate. Generally, we argue that Q is not a physically justified property of the medium, and that viscoelastic models formulated in terms of "$Q(f)$ spectra" do not meet the standard of a physical theory (Morozov, 2009c and in review). Thus, this paper only deals with Q as an empirical quantity. Similarly to Morozov (2008 and 2010a-b), we can only ascertain a frequency-independent body-wave Q of the Earth, albeit with a different meaning of Q denoted by Q_e below. This frequency independence was even presented as our principal goal in a critique by Xie (2010). However, a constant-Q_e model is by no means intended by the present analysis. In fact, theoretical studies (*e.g.*, Biot, 1956; Kbopoff and MacDonald, 1958; Morozov, 2010d, and in review) show that a constant Q_e is difficult to achieve from physical principles, and a perfectly frequency-independent Q is also ruled out by causality (Futterman, 1962). Nevertheless, an *arbitrary* frequency-dependent Q is similarly difficult to achieve, because physical models predict only definite dependences (Morozov, 2011b and in review). Our model therefore expects a generally frequency-dependent Q_e, but within the available data bandwidths and quality, a constant Q_e still appears to be all that can be reliably measured. Hence, we retain such a Q_e as an empirical attribute. When multiple wave types with different Q_e are combined across a very broad frequency band, an apparent "absorption band" re-appears as a scaling phenomenon (Morozov, 2010b).

2 Attenuation coefficient, t^*, and Q

Let us consider the general amplitude-decay problem for a propagating or standing wave without relying on any knowledge about the details of propagation. In seismic attenuation observations, some type of a amplitude is usually measured as a function of frequency and time, $A(f,t)$. The meanings of A and t here may be different for different observations. For example, for coda waves, they correspond to continuous measurement of time-averaged amplitude at a single receiver, but for body waves – to following a selected wave onset propagating in space. When corrected for all known (or inferred) effects of the sources, receivers, and background propagation (such as GS and maybe scattering and attenuation expected in a known structure), this amplitude becomes the "path effect", $\delta P(f,t)$. Once again, this "path" may actually involve propagation of different waves along multiple paths, such as, for example, in a seismic coda. In a perfectly known Earth's structure, δP would remain constant for all $t > 0$: $\delta P(f,t) \equiv \delta P(f,0)$, but with a small inaccuracy in the model, it will gradually deviate from this level with increasing time:

$$\delta P(f,t) = \delta P(f,0)\exp\left[-\chi^*(f,t)\right], \tag{1}$$

where $\chi^*(f, t=0) = 0$. The absolute value of χ^* increases with time, and for relatively short times, it can be further approximated as proportional to t:

$$\chi^*(f,t) = \chi(f)t \; . \quad (2)$$

This is the usual perturbation- (or scattering-) theory approximation. We will refer to both χ^* and χ in eqs. (1) and (2) as the generalized attenuation coefficients, which include the effects of GS, elastic/inelastic losses, and inaccuracies of the background model.

Further, let us consider the frequency dependences of each of these χ. Denoting the zero-frequency limits in these quantities by $\gamma = \chi|_{f\to 0}$, we obtain:

$$\chi^*(f,t) = \gamma^* + \frac{\pi}{Q_e^*} f \text{ , and } \chi(f) = \gamma + \frac{\pi}{Q_e} f \; , \quad (3)$$

where Q_e is the "effective attenuation" quality factor, and quantities with asterisks refer to the corresponding time-average properties. For body waves, the time averages usually become path averages. Note that the above expressions are merely mathematical identities isolating the zero-frequency limits in χ and χ^*, and both Q_e and Q_e^* are generally frequency-dependent. Also note that γ and Q_e^* are measured in frequency units.

In conventional measurements of seismic attenuation, attenuation coefficients (1) and (2) are not considered directly but replaced with parameters t^* and Q defined as (*e.g.*, Der and Lees, 1985; Aki and Chouet, 1975, respectively):

$$t^*\left(f,t\right) = \frac{\chi^*(f,t)}{\pi f} \text{, and } Q^{-1}\left(f\right) = \frac{\chi(f)}{\pi f} \; . \quad (4)$$

Expressions (4) implicitly assume that both χ's must tend to zero at $f \to 0$. Also, frequency dependences of these quantities are often sought in the power-law form of $t^* \propto Q^{-1} \propto f^{-\alpha}$, where α is between 0.15–0.4 (Anderson and Minster, 1979). Thus, from the outset of the conventional model, it is *postulated* that:

$$\gamma^* = 0, \; \gamma = 0, \text{ and } Q_e^* \propto Q_e \propto f^{\alpha} \; . \quad (5)$$

The first two constraints in (5) are very strong assumptions, which facilitate the inversion for $Q(f = 1 \text{ Hz})$ and α from many amplitude-attenuation datasets. However, considering the raw frequency dependences of χ and χ^*, one can see that these constraints are usually violated, and the data only suggest non-zero γ and γ^* and constant Q_e and Q_e^* within the available frequency bands (Morozov, 2008, 2010a,b, and 2011b).

In contrast to (4) and (5), our approach (3) simply consists of: i) *allowing* non-zero limits of χ and χ^* at $f \to 0$, and ii) considering the simplest functional dependence of $\chi^*(f)$ and $\chi(f)$ from this limit, *i.e*., taking a Taylor series near $f = 0$. Eqs. (3) then suggest a straightforward approach to analyzing the observed $\chi^*(f)$ and $\chi(f)$ dependencies. The key to interpreting attenuation coefficients is in examining them for: 1) non-zero intercepts (even if extrapolated to $f \to 0$ based on the available trends), and 2) linearity in respect to f within the available frequency bands. If such linear intervals are found, they suggest a stable wave-mode content and frequency-independence of attenuation within these bands. From linear approximations (3), the intercept values (γ or γ^*) can be interpreted as parameters of the residual GS (including scattering; Morozov, 2011a), and Q_e^{-1} or Q_e^{*-1} – related to the attenuation properties of the medium. Note that although an assumption about the residual GS being frequency-independent still has to be made, it only affects the separation of the generalized GS and Q_e^{-1} effects. Such separation is indeed ambiguous and conceptually uncertain (Morozov, 2010a).

The approximation (1)– (3) represents only a perturbation model of seismic amplitude measurements and is *model-independent* in respect to the GS or other models of elastic-wave propagation. This approximation makes no assumptions about the background seismic amplitude spreading, is exact in f, and is only limited in time by the requirements $|\chi| t \ll 1$ and $|\chi^*| t \ll 1$ (Morozov, 2010a). Quantities χ and χ^* only measure the deviations of the observed amplitudes from those predicted by the background model, and Q_e and Q_e^* denote the frequency-dependent parts of these deviations. Requirement $|\chi| t \ll 1$ represents the only limitation of this approximation, which can sometimes be violated (such as in Appendix 1 by Xie's (2010), who used too long observation times) but can always be corrected by using a more accurate background model. Thus, for body waves, Q_e simply measures the frequency-dependent amplitude decay which cannot be attributed to the best-known effects of the structure. This appears to be a reasonable approximation for the attenuative property of the Earth's material.

Returning to the conventional parameters (4), we obtain from eqs. (3):

$$t^*(f,t) = \left(Q_e^*\right)^{-1} + \frac{\gamma^*}{\pi f}\text{, and } Q^{-1}(f) = Q_e^{-1} + \frac{\gamma}{\pi f}\,. \tag{6}$$

With Q_e = const, the second of these formulas was used by Dainty (1981) to describe the *S*-wave $Q^{-1}(f)$ at 1–30 Hz. Dainty (1981) interpreted the term $\gamma/\pi f$ as "scattering Q^{-1}" and noted its characteristic f^{-1} dependence. However, as shown in Morozov (2008, 2010a, d), scattering can hardly be unambiguously isolated in seismic-amplitude data. We therefore interpret γ as an empirical "residual GS," which also incorporates the effects of small-scale scattering as those of the "stochastic" GS. Taking the limit of $f \to 0$ in eq. (1), we see that the residual-GS correction has the form of $\exp(-\gamma t)$.

In media with non-trivial structures causing $\gamma \neq 0$ (such as containing short-scale heterogeneity or additional ray bending; cf. Morozov, 2011a), both t^* and Q in eqs. (4) exhibit spurious variations with frequency. For example, when $\gamma > 0$, the apparent t^* decreases, and Q increases with frequency. Such faster-than-t^{-1} GS was noted long ago and explained by the presence of the downward-reflective upper crust (Frankel *et al.*, 1990; Morozov, 2010a). Positive values of $\gamma \approx$ 4–60 mHz appear to be typical for crustal body, coda, and surface waves (Morozov, 2008, 2010b), and similar cases are discussed below.

In addition to t^*, which is measured from GS-corrected amplitudes, Der and Lees (1985) defined the "apparent t^*", here denoted by $\bar{t}^*$:

$$\bar{t}^* = -\frac{1}{\pi}\frac{\partial}{\partial f}\left(\ln \delta P\right) = \frac{1}{Q_e^*}\left(1 - \frac{\partial \ln Q_e^*}{\partial \ln f}\right). \tag{7}$$

This quantity is determined from spectral ratios and is independent of γ. Comparing eqs. (6) with (7), note that the relative difference between t^* and $\bar{t}^*$ is also caused by the GS factor (γ) and is inherently frequency-dependent:

$$\frac{t^* - \bar{t}^*}{\bar{t}^*} = \frac{\dfrac{\gamma^* Q_e^*}{\pi f} + \dfrac{\partial \ln Q_e^*}{\partial \ln f}}{1 - \dfrac{\partial \ln Q_e^*}{\partial \ln f}} \approx \frac{\gamma^* Q_e^*}{\pi f}. \tag{8}$$

This ratio is related to the t^*-bias function by Der and Lees (1985).

Another useful way to understand the effects of the residual GS on t^* measurements is in relating $\bar{t}^*$ to t^* (Der and Lees, 1985):

$$t^* + f\,\frac{dt^*}{df} = \bar{t}^*. \tag{9}$$

If the apparent $\bar{t}^*$ is known, this differential equation can be integrated to obtain the "true" $t^*(f)$, as done by Der and Lees (1985). However, this integration is non-unique, and its uncertainty is given by the solution to the homogeneous counterpart of equation (9). This solution reads $t^* = af^{-1}$, where a is an arbitrary constant. By taking $a = \gamma^*/\pi$, we see from eq. (4) that this homogenous solution again corresponds to the residual GS.

3 Frequency dependence of mantle body-wave *t**

As shown above, both the $t^*(f)$ and $Q(f)$ descriptions represent special cases of the general attenuation coefficients (2) and (3). By their definitions (4), these cases require a perfect model for background wave propagation. In the presence of variable and poorly known GS (*i.e.*, common observational case), the general $\chi(f)$ form appears to be most suitable and least prone to uncertainties.

Below, we show how the $\chi(f)$ approach changes some of the existing t^*-based interpretations. Although the attenuation coefficients $\chi^*(f)$ and/or $\chi(f)$ should ideally be measured directly from the raw wave-amplitude data, they can also be estimated from the reported t^* and Q values by inverting eqs. (1) and (4):

$$\chi^* = \pi f t^*, \text{ and } \chi = \frac{\pi f}{Q}. \tag{10}$$

Fig. 1a shows the $\bar{t}^*(f)$ and $t^*(f)$ data summary from Der *et al*. (1986b), derived from a series of studies analyzing multiple *P*- and *S*- body-wave phases at 25–90° source-receiver ranges, with paths lying within the shield areas of Eurasia. In addition, the $\bar{t}^*$ and t^* dependencies predicted by ray tracing in a layered, frequency-dependent Q model EURS are shown by dashed lines (Fig. 1a). An earlier shield-path model (Der *et al*., 1982; grey dotted line in Fig. 1a) is also overlain on this plot. This model is also close to the mantle absorption band model by Minster (1978a, b).

Taking an $\chi^*(f)$ point of view, we see that the same $\bar{t}^*$ and t^* data can also be explained by a linear dependence (3) with $\gamma^* \approx 0.06$ and $Q_e^* \approx 5.5$ Hz (thick solid line in Fig. 1a). Note that this line fits the GS-independent data (*P*-wave spectral measurement within 1–10 Hz range) even better than the existing model, and it may also be better following the trend of t^* rising at the lower frequencies. A marginal fit of the multi-phase *S* and poor fit of the rise-time data (Fig. 1a) could be due to poorer reliability of these techniques, and it was similarly problematic in the original interpretation by Der *et al*. (1986a). Difficulties in fitting multiple data types and impracticality of formal inversion were noted in several t^* studies (*e.g*., Der *et al*., 1986a,b; Sharrock *et al*., 1995).

Fig. 1b illustrates a simple interpretation technique that was used to analyse the t^* data above. By using eq. (10), t^* values were first transformed into χ^*/π, and then trial "reductions" (subtractions of terms *af* with variable *a*) were applied until the resulting data distribution appeared near horizontal (Fig. 1b). This procedure gave the optimal intercept (γ^*) and slope (π/Q_e^*) of the $\chi^*(f)$ line. Note that a curve with a slight "absorption band" (increased χ^* from ~0.1 to ~ 5 Hz, similarly to the dashed black curve in Fig. 1b) might fit the data a little better; however, this detail still does not seem to be warranted by the scatter in the data (Der *et al*., 1986b).

With some concerns about the data fit and possible mild frequency dependence of Q_e in the middle of the long-period band, the principal result remains clear: a positive shift of $\gamma^* \approx 0.06$ in GS can explain the observed increase of t^* from ~0.2 s at 1–10 Hz to ~1–2 s at 0.01–0.02 Hz (Fig. 1a). The effective attenuation is practically frequency-independent and equals $Q_e^* \approx 5.5$ Hz, which corresponds to $t_e^* = 1/Q_e^* \approx 0.18$ s. Note that the level of $\gamma^* \approx 0.06$ corresponds to only a ~6% shift in GS compensation relative to the assumed level of $\delta P(t,f) = 1$ (eq. (1); Fig. 1a). Such bias in the forward model should be expected from the crustal or uppermost-mantle structural effects (section 5), and it should definitely not be presented as a frequency dependence of mantle *Q*.

The above analysis shows that similarly to *Q*(*f*), the traditional t^* is an "apparent" quantity which trades off with the GS correction, *i.e*. with the measurement procedure. Values of t^* change wherever the real GS differs from the postulated theoretical level. By contrast, the $\bar{t}^*$ termed "apparent" by Der and Lees (1985) is measured from the spectral ratios and is therefore closer to the "true" Q_e^* property of the Earth (eq. 7). For this reason, spectral-ratio measurements should be significantly more reliable in constraining the *in situ* attenuation.

Based on the above property of $\bar{t}^*$, a simple technique for GS-assumption independent interpretation of attenuation is shown in Fig. 2. Instead of deriving t^* from $\bar{t}^*$ by an ambiguous integration of eq. (9), we can determine the $\bar{t}^*$ from reported t^* data by using the same equation. The empirical dt^*/df trend for use in eq. (9) can be estimated from the same t^* data plot (Fig. 2a). Once again, similarly to Fig. 1a, rise-time data are poorly consistent with the other datasets. The resulting $\bar{t}^*$ values simulate spectral measurements within the long-period band, and they are independent of the uncertainty in the GS. Therefore, $\bar{t}^*$ values can be reliably interpreted and modeled. For the present data, they can be satisfied with a near-constant $\bar{t}^* \approx 0.18$ s value mentioned above (Fig. 2b).

Our derivation of χ^* from published t^* data still inherits all limitations and approximations of these data, such as: 1) the *P*- and *S*-wave data are tied together by simple scaling $t_S^* = 4t_P^*$, 2) t^* are considered to be independent of *t*, 3) the same GS rate γ^* is assumed for *P*- and *S*-waves, their reflections and multiples, and 4) the free-surface refection coefficient for ScS_n multiples is assumed to equal 1 (Der *et al*., 1986a,b; Lees *et al*., 1986). None of these limitations are necessary in the attenuation-coefficient approach, and the corresponding parameters can be included in the model. Inverting the raw data spectra, amplitudes, and pulse shapes directly within the χ^* model would reduce the errors and improve the data fit (Fig. 1) and quality of our interpretation. However, this would require revisiting large, old and complex datasets, which are (unfortunately) not available to the present analysis.

4 Velocity dispersion

Important constraints on long-period *P*-wave attenuation within the mantle and inner core were derived from time-domain velocity dispersion measurements (Cormier, 1982; Doornbos, 1983; Li and Cormier, 2002; Savage *et al*., 2010). However, two significant problems with relating this dispersion to attenuation have received little attention so far: 1) the velocity dispersion is always viewed as only caused by *Q*, and 2) this dispersion relation is always inferred from the Kramers-Krönig identities. Nevertheless, these identities contain *no specific information about the physics of attenuation*, but only express the requirement of causality that should be satisfied by all traveling waves. For example, the dispersion can be caused by diffractions on the Earth's inner structure and unrelated to *Q*. Another important caveat of the Kramers-Krönig relations, also often unnoticed (*e.g*., Dahlen and Tromp; also see eq. (A7) in Appendix A), is that they only apply to the wave solutions and not to the elastic moduli of the medium. Velocities $V(\omega)$ and apparent quality factors $Q(\omega)$ in the next paragraph are properties of the seismic phase and not the wave speed (*i.e*., a combination of elastic moduli with density) of the medium and its attenuation properties.

Despite the above critique, let us nevertheless try using the traditional relation of attenuation to velocity dispersion to estimate the amount of time delay experienced by long-period body waves relative to short periods. This delay is known to be near constant at teleseismic ranges and equals ~1 s (Fig. 3; Doornbos, 1983).

For waves in a homogenous medium, arrival-time delays can be related to phase-velocity variations with frequency, and for a frequency-independent *Q*, and Kramers-Krönig relations show that the plane-wave phase (V) and group velocities (U) should increase with frequency as (eq. (A8) in Appendix A):

$$\delta\left(\ln U\right) \approx \delta\left(\ln V\right) \approx \frac{1}{\pi Q}\delta\left(\ln\omega\right). \qquad (11)$$

Again, eq. (11) should not be read it as "velocity depending on the frequency." Neither V, dispersion rate $\delta(\ln V)/\delta(\ln\omega)$, nor Q physically depend on frequency, but all three of these quantities are determined by the elasticity and internal friction within the seismic wave. A trivial one-to-one relation of V to Q, as in eq. (11), only arises in the case of a uniform medium, in which the attenuation reduces to a multiplication of the amplitudes by $\exp\left[-\omega/2\left(VQ\right)\right]$. Taking expression (11) as an *ad hoc* estimate of dispersion in the absence of an accurate solution, we can estimate the associated travel-time delay at long periods: $\delta t_D \approx -t\delta\left(\ln U\right)$, where t is the travel time. Integration of eq. (11) over travel time shows that this delay is proportional to t^*:

$$\delta t_D \approx -\frac{\overline{t}^*}{\pi}\delta\left(\ln\omega\right). \qquad (12)$$

An important question arises whether the Q and t^* in eqs. (11) and (12) should be the total frequency-dependent viscoelastic Q and t^* or the effective Q_e and $\overline{t}^*$ defined above. As shown in Appendix A, the Q in eq. (11) represents the slope of $d\chi/d\omega$ near $\omega = 0$, and therefore it should correspond to Q_e. Accordingly, t^* in eq. (12) should be the GS-corrected $\overline{t}^*$, as also shown there. With the value of $\overline{t}_P^* \approx 0.18$ s measured above, eq. (12) gives $\delta t_D \approx 0.4$ s for the frequency range in Fig. 2.

Thus, the attenuation within the low-Q_e uppermost mantle could account for about a half of the dispersion-related long-period travel-time delay discussed by Doornbos (1983). Regarding the remaining 0.6 s of delay (Fig. 3), we can only make several general comments. First, for heterogeneous Earth, expressions (11) and (12) represent only heuristic estimates of dispersion, and they should be updated in a more rigorous solution. Second, for long waves at 20–40-s periods, the geometric spreading (propagation in the absence of energy absorption or small-scale scattering) within the upper mantle should be frequency-dependent (*e.g.*, Young *et al*., 2007), which would cause a dispersion effect as well. Some frequency-dependent delay can be caused by P/S mode conversions on the base of the crust and lithospheric layering. Finally, the earthquake source function may also need to be examined for possible delays with respect to the modelled signatures. An "absorption band" in the mantle Q^{-1} is of course also possible, but compared to the above or other potential explanations, it appears phenomenological and relatively schematic.

5 *In situ* attenuation within the upper mantle

Der and McElfresh (1980) noted that body-wave values of $t_P^* \sim 1$ s and $t_S^* \sim 4$ s could not be constant for all frequencies and paths, and also pointed out the regional variability of the short-period t^*. They concluded that t_P^* and t_S^* should be much lower within the short-period band, because otherwise the energy at 4–5 Hz would not be observed in P waves. Such frequency dependence of t^* is therefore well established. Nevertheless, this t^* is still an apparent quantity, and its (usually assumed) relation to the Earth's *in situ Q*:

$$t^*(f) = \int_{\text{Ray path}} Q^{-1}(f)\,dt\,, \tag{13}$$

is incorrect. This expression automatically projects the frequency dependence of t^* into the *in situ* Q, whereas, as shown above, t^* contains contributions from the residual GS, which increases at long and intermediate periods.

To characterize the range of frequencies at which the residual GS effects in eq. (13) are significant, it is convenient to use the "cross-over" frequency $f_c^* = \gamma^* Q_e^*/\pi$ (Morozov, 2008). Below this frequency, the effects of GS dominate those of Q_e^{-1}. For the data in Fig. 1b, $f_c \approx 0.1$ Hz. This shows why t^* values are stable for the short-period data ($f >> f_c$) but quickly increase at frequencies near or below this level.

A replacement for the path integral (13), which is stable with respect to the GS uncertainty, arises from expressions (1) and (2):

$$\chi^* = \int_{\text{Wave path}} \chi_i\,dt\,, \tag{14}$$

where χ_i is the "intrinsic", or "local" attenuation coefficient. The meaning of the "wave-path" integration may be variable for different types of waves, which can likely be informally described by Feynman path integrals (Morozov, 2010b). By isolating its zero-frequency limit, χ_i can also be decomposed similarly to eq. (3):

$$\chi_i(f) = \gamma_i + \frac{\pi}{Q_i} f\,. \tag{15}$$

The effect of γ_i should be the logarithm of the residual GS in the actual velocity/density structure. For several theoretical models of this property, see Morozov (2011b). Because γ_i physically describes the additional ray curvature or small-scale scattering in access of that produced by the background model, this parameter should be related to the local velocity gradients and reflectivity within the Earth's structure. Note that similarly to the velocity gradients and mean reflection coefficient, γ_i is measured in frequency units.

For shorter-period body waves, we can take the wave-path integration in eq. (14) as occurring along approximately frequency-independent rays, and consequently the "geometric" term also obeys a similar equation:

$$\gamma^* = \int_{\text{Ray path}} \gamma_i dt \,. \tag{16}$$

Taking into account eq. (7), the apparent $\overline{t}^*$ (but not t^*!) can also be expressed by a similar path integral:

$$\left(Q_e^*\right)^{-1} = \overline{t}^* = \int_{\text{Ray path}} Q_i^{-1} dt \,. \tag{17}$$

Thus, γ_i behaves similarly to χ_i and Q_i^{-1}; in particular, path integrals of both of γ_i and Q_i^{-1} are nearly constant at teleseismic distances. As noted by Der and Lees (1986), high attenuation (Q_i^{-1}) is concentrated within the uppermost mantle, leading to $\overline{t}_P^* \approx 0.2$ s for all source-receiver ranges beyond ~25°. Similarly, the zone of high velocity gradients (and therefore γ_i) is also localized in the uppermost lithosphere and explains the finite and range-independent value of $\gamma^* \approx 0.06$ (Fig. 1b). Interestingly, this value of γ^* appears to be accumulated within a fairly thin layer, likely the crust or crust-mantle transition. This conclusion follows from the values of the differential body-wave γ in the range of 4– 20 mHz (with a maximum of 8 mHz in stable areas; Morozov, 2008), with which it only takes 3–15 s of propagation time to produce the teleseismic γ^*. As expression (14) also suggests, values of γ^* should increase within tectonically-active areas, which are characterized by higher γ_i (Morozov, 2008, 2010a, b). This prediction agrees with larger t^* observed in tectonically-active areas (Sharrock *et al.*, 1995).

6 Newer arguments for mantle $t^* \propto f^{-\alpha}$

In this study, we attempted a new look into some aspects of several older datasets (Der *et al*., 1982, 1986a,b; Lees *et al*., 1986; Doornbos 1983). Using such "classic" data offers several advantages. First, these data are broadly known and represent key arguments in the discussion of mantle properties. They were also produced at the time when the concept of frequency-dependent Q was only being established, and data presentations were argumentative and close to the first principles of seismic observations. These data were also interpreted and reported in relatively straightforward ways, with clear connections between the underlying models and conclusions. Because of this, these datasets can still be reviewed and understood now, and new and instructive interpretations produced. Acquisition of new data should not affect the conclusions made from these datasets.

Modern datasets are much larger, digital, better quality, and their processing and presentation emphasizes more sophisticated numerical techniques. However, largely because of the same improvements, "model footprints" in them can be much more difficult to identify and isolate. Many inversions are now conducted in corroboration of the idea of the mantle Q varying (typically increasing) with frequency. The power-law exponent α in $Q^{-1} \propto f^{\alpha}$ law is sometimes derived even without constructing spatial models for Q. For example, Lekić *et al*. (2009) inferred the values of α for the Earth's mantle by manipulating Fréchet kernels of the forward problem and projecting the long-period apparent-Q data with these kernels. Even though not looking for a model which would fit the data and despite the existence of "older", detailed models showing $\alpha = 0$ (such as model QL6; Durek and Ekström, 1996), these authors nevertheless argued that $\alpha \neq 0$ was reliably constrained. This is an example of the "model footprint" discussed above.

Another important recent study suggesting $\alpha > 0$ for body waves within the mantle was reported by Shito *et al*. (2004). These authors used shorter-range (~3–15°) P-wave recordings at 0.08–8-Hz from intraslab earthquakes in Japan. They also derived no spatial attenuation model but inferred the values of α and differential t^* (denoted δt_0^*) from stacked spectral ratios for different pairs of stations. For three levels of t^*, these frequency-dependent trends at ~3-Hz frequencies are shown by white arrows in Fig. 1a. Several assumptions were made in this derivation (*ibid*): 1) frequency-independence of geometric spreading, 2) absence of attenuation ($Q^{-1} = 0$) within the lithospheric slab, 3) negligible receiver effects on spectral ratios, and 4) insignificance of noise. All of these assumptions are questionable, and it is also impossible to evaluate this interpretation without completely redoing the data analysis. However, the observed correlation between the values of δt_0^* and α still looks suggestive of a trade-off (Fig. 4a). Constructing the attenuation coefficients (4) from Shito's *et al*. (2004) frequency-dependent $\delta t^* = \delta t_0^* \left(f / f_0 \right)^{-\alpha}$, we obtain: $\chi^*\left(f \right) = \pi f_0 \delta t_0^* \left(f / f_0 \right)^{1-\alpha}$ (with f_0 = 1 Hz). The recorded spectra are stationary near ~5 Hz (see Fig. 2 in that paper), and so by levelling the average values of $\chi^*\left(f \right)$ at this frequency, we can superimpose all of the spectral ratios predicted by this model (Fig. 4b). This plot shows that almost all $\chi^*\left(f \right)$ values fall within a single standard deviation (error bar and dashed lines in Fig. 4b) from a common linear trend. Note that the values of α and δt_0^* correlate with the depths of the sources and also trade off positively (*ibid*). This means that if attenuation (δt_0^*) increases with depth (which appears likely), it could also cause a correlated positive bias in α and δt_0^* (dashed line in Fig. 4a). Additionally, the concave shapes of $\chi^*\left(f \right)$ at low frequencies (arrow in Fig. 4b) could be partly caused by the low-frequency noise, likely coming from oceanic waves (see Fig. 2 in Shito *et al*., 2004). Thus, the resulting inference of a power-law dependence for mantle *Q* may again be influenced by the selected model itself.

Along with contrasting t^* values arising from long- and short-period body-wave measurements, a completely different argument is sometimes advanced in favour of the mantle *Q* increasing with frequency, namely the observations of a high-frequency "teleseismic P_n" in Peaceful Nuclear Explosion (PNE) profiles in Russia. Ryberg and Wenzel (1999) found that PNE waves at frequencies above 5 Hz traveled at P_n velocities to over 3000-km distances within the uppermost mantle of the East European Platform. They further proposed a multiply-scattering, anisotropic waveguide favouring preferential propagation of high-frequency waves within the upper 80 km of the mantle. It is still unclear whether such strong scattering would actually increase or decrease the uppermost-mantle *Q* at these frequencies.

However, the scattering-waveguide interpretation of the high-frequency teleseismic P_n is yet another notable example of mistaking structural effects for a frequency dependence of mantle attenuation. A detailed analysis shows that the teleseismic P_n from the PNEs consists of a series of surface *P*-wave multiples traveling above the depth of ~100 km, below which a strong increase in attenuation is present (Morozov *et al*., 1998a,b). Consequently, the high-frequency (up to ~10–15 Hz) amplitudes of these waves appear anomalously strong only when compared to the teleseismic *P* waves penetrating the attenuative layers below ~150–220-km depths (Morozov, 2001). The high-frequency *P* waves and their multiples travel relatively efficiently in this area merely because of the low attenuation within the mantle lid ($Q \approx 1400$–2000; Morozov *et al*., 1998b). Pronounced codas of the teleseismic P_n arrivals were explained by crustal scattering (*e.g*., Nielsen *et al*., 2003).

7 The inner core

Seismic attenuation within the inner core is a very extensive subject, and we only offer two remarks inline with the preceding discussions. First, the measured dispersion $\delta(\ln V)/\delta(\ln\omega)$ is about (0.2–0.6)·10^{-4} (Li and Cormier, 2002), which is significantly lower than ~3·10^{-3} predicted by eq. (12). Such weak dispersion was interpreted as an indication of an absorption band (Doornbos, 1983; Li and Cormier, 2002). However, such low velocity variation means that the relative time delay from 1-s to 25-s periods is only $\delta t_D/t \approx (0.6\text{–}2.0)\cdot 10^{-4}$. This effect is much smaller than the uncertainties of any of the models involved in its estimation, and it could be caused, for example, by frequency-dependent geometric spreading. Therefore, additional research is required for corroborating the association of this delay with an absorption band within the inner core.

Second, the available *PKIKP* t^* values derived from spectral-ratio measurements (Fig. 5a; data and interpretation from Doornbos, 1983) show a variation with frequency opposite to that of the upper-mantle body waves (Fig. 1a). Doornbos (1983) suggested that such behaviour was due to the seismic band being located within the *lower* flank of the absorption band, so that Q^{-1} and t^* increase with frequency. Generally, this observation appears to be correct, although this “absorption band” may be not of the relaxation-based type assumed in the viscoelastic model.

As Fig. 5b shows, an increase in t^* arises from χ^* increasing non-linearly with frequency. Interestingly, the trend of these $\chi^*(f)$ resembles that of normal modes (Morozov, 2010b), suggesting that it may potentially be affected by the sphericity of the Earth and its velocity/density distribution. A linear $\chi^*(f)$ model with $\gamma^* < 0$ (similarly to surface waves, Morozov, 2010b), is also possible, in which case the attenuation would be twice stronger: $\left(Q_e^*\right)^{-1} \approx 2t^*\left(1\ \text{Hz}\right)$ (dashed and dotted lines in Fig. 5b, and eq. (6)). Modeling these observed t^* values would require accurate corrections for elastic and velocity-model related effects (Doornbos, 1983), which are not clear at present.

Leaving the background model aside, also note the remarkable similarity of the inner-core $\chi^*(f)$ to the frequency dependence of the attenuation coefficient in Biot's (1956) model of fluid-saturated porous rock (Fig. 5c). An accelerating increase in χ rather than a constant Q should be the norm to which other cases should be compared. Attenuation (either anelastic or scattering Q^{-1}, or t^*) increasing with frequency "naturally" arises in realistic mechanical systems, in which the heterogeneity and "internal mobility" of various kinds usually increases at smaller scales. Once again, a specific mechanical model of rhelogy within and near the inner core is required. A simple Lagrangian model of this rheology descending from the early studies by Knopoff and MacDonald (1958) was recently given by Morozov (in review). The absorption band concept gives only a phenomenological replacement for such a model, and it may be not particularly helpful in discovering the physical mechanisms of attenuation.

8 Discussion

As shown above, Q_e is practically frequency-independent from the present mantle body-wave data (Fig. 1), and after a ~6% GS correction, long-period t^* values become close to those observed at short periods ($t^* \approx 0.18$ s for P waves at ~25° ranges). From eq. (17), this means that all the data in Fig. 1 can be explained by a frequency-independent Q of the mantle. Inversion for the depth and regional Q_i variations is a complex problem impregnated with further difficulties, and it will be addressed elsewhere. However, from the values of γ^* and Q_e^*, and eqs. (14) and (17), it is clear that the *in situ* Q model *can* be frequency-independent unless additional data provide evidence to the contrary.

Interestingly, it appears that *Q increasing with frequency* tends to frequently arise from various uncertainties and limitations of seismic data analysis. From sections 2 and 6, as well as from Morozov (2008, 2010a, b, 2011b, and in review) and Morozov *et al*. (2008), very different factors lead to the apparent $Q(f)$ increase: 1) faster-than modelled geometrical spreading (caused, *e.g*., by diving and bending rays), 2) scattering and reflectivity in layered structures, 3) background noise during measurements, 4) differences in the scale-lengths of observations, with shorter-scale oscillations (*e.g*., concentrated at shallower depths) often exhibiting higher Q's, and 5) non-linear rheology and thermoelasticity. Such preference for an $\alpha > 0$ when looking for a universal $Q \propto f^{\alpha}$ law can be explained by looking at the attenuation coefficient: $\chi \propto f^{1-\alpha}$. Each of the above factors cause χ to increase with frequency somewhat slower than f, generally because the oscillation frequency f is not the only factor affecting energy dissipation.

Recognition of only a frequency-independent Q within the mantle may appear too simple and discordant with numerous arguments in favour of the frequency-dependent Q. However, it is important to clearly realise the limits of the observational evidence and differentiate between the truly constrained quantities and artefacts resulting from simplifying assumptions and mathematical models. In particular, the physical meaning of the *in situ* seismological Q still needs to be clarified. As demonstrated in this paper and in Morozov (2008, 2009a,b, 2010a–d), a background-model independent analysis shows a spatially-variable GS and Q but no indications of their frequency dependences. Note that a frequency-independent Q_e represents a significantly stronger constraint than permissive frequency-dependent models. With many assumptions and internal parameters, depth- and frequency-dependent Q models become capable of explaining nearly any data and may be virtually unverifiable.

In numerous recent publications on seismic attenuation, the following general type of conclusion can be found: "…assuming a uniform and isotropic background, no free surface, geometric spreading of $1/r$, and using some standard mathematical methods (single-scattering, energy flux, *etc.*), it is found that thus-defined Q increases with frequency as (for example) $f^{1.07}$…" While being accurate and recognising all the necessary assumptions, such results can only be understood as parametric descriptions of the data, *i.e*. frequency-dependent wave amplitudes in the study area. However, as we know that most of these assumptions are inaccurate within the error limits required to measure the $Q(f)$, it is hard to say how such Q is related to the properties of the Earth. The *in situ* character of Q remains the principal but untested assumption. By contrast, our description above focuses strictly on quantitative estimates made from the data and on measuring the effects of model inaccuracies.

In studies pursuing purely phenomenological models, such as earthquake engineering and nuclear-test monitoring, the physical character of attenuation may be insignificant, and only accurate predictions of seismic amplitudes are sought. These predictions can be achieved by applying various conventions and empirical rules, for which the above discussions of the frequency-dependent Q is both insufficient (in terms of accuracy) and redundant (in content). However, even in such applications, it appears that compared with expression (3), the conventional parameterization (4) is insufficient (lacking the $f \to 0$ limit) and consequently prone of instabilities. Compared with (3), this parameterization is also too elaborate mathematically, which seems to be unnecessary in empirical studies.

Conclusions

Similar to Q measured from surface-, body-, and coda waves, frequency-dependent t^* values observed in long- and short-period P- and S-wave studies represent apparent quantities. As in the Q case, inaccurate assumptions about the geometric spreading (GS) may be responsible for the observed increase of t_P^* from ~0.2 s at short periods to ~1–2 s at long periods. Using a perturbation-theory, attenuation-coefficient formulation, this residual GS is estimated as positive and equal ~6% of the theoretical background level. The frequency- and range-independent bias in the GS is caused by a cumulative effect of the lithospheric velocity heterogeneity, similarly to the accumulation of a nearly-constant t^* within the same ranges. This bias appears to be accumulated within a relatively thin part of the lithosphere, likely within the crust or even the upper crust in active tectonic zones. Correction for this residual GS removes the frequency dependence of body P-wave t^* and makes all teleseismic $t_P^* \approx 0.18$ s at all frequencies. Therefore, the *in situ* upper-mantle Q may be frequency-independent within the available data constraints and slightly greater than its current short-period levels. Velocity dispersion corresponding to this level of attenuation explains about 40% of the observed delay of long-period body waves. To explain the remaining long-period delay, a better model for dispersion is required, including mantle heterogeneity and the physics of energy dissipation. For the inner core, the Q^{-1} (more precisely, the slope of the attenuation coefficient, $d\chi/df$) increases with frequency, which is similar to the one predicted by poroelasticity. Further explanation of this increase also requires a detailed physical theory.

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Figures

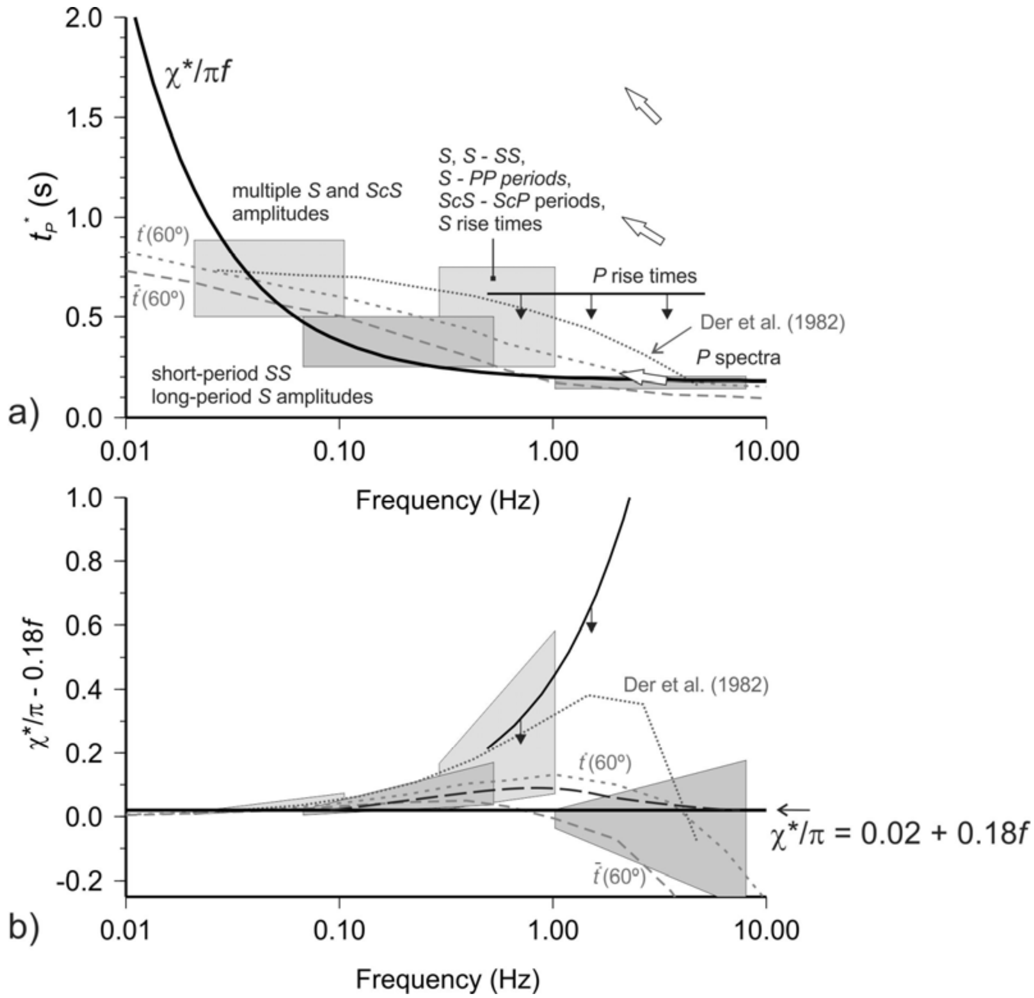

Fig. 1. Interpretation of t^* data in Eurasia (Der *et al*., 1982, 1986a,b; Lees *et al*., 1986) by using the attenuation coefficient χ^*:

a) Data summary from Der *et al*. 1986) with $\bar{t}^*$ and t^* curves modeled by ray tracing at 60° ranges in their EURS Q model (grey dashed lines). Different data sources, measurement methods, and frequency-t^* value ranges are indicated. The $t^*(f)$ curve for shield areas from Der *et al*. (1982) is shown by grey dotted line. Thick black line corresponds to the attenuation coefficient linear in frequency, with $\gamma^* \approx 0.06$ and $Q_e^* \approx 5.5$ (eq. 3). White arrows illustrate the $t^* \propto f^{\alpha}$ dependences for three levels of t^* at $f = 3$ Hz and $\alpha = 0.3$.

b) The same data in the form of "reduced" attenuation coefficient. Dashed black line shows an alternate interpretation with slightly frequency-dependent Q_e^* or γ^*.

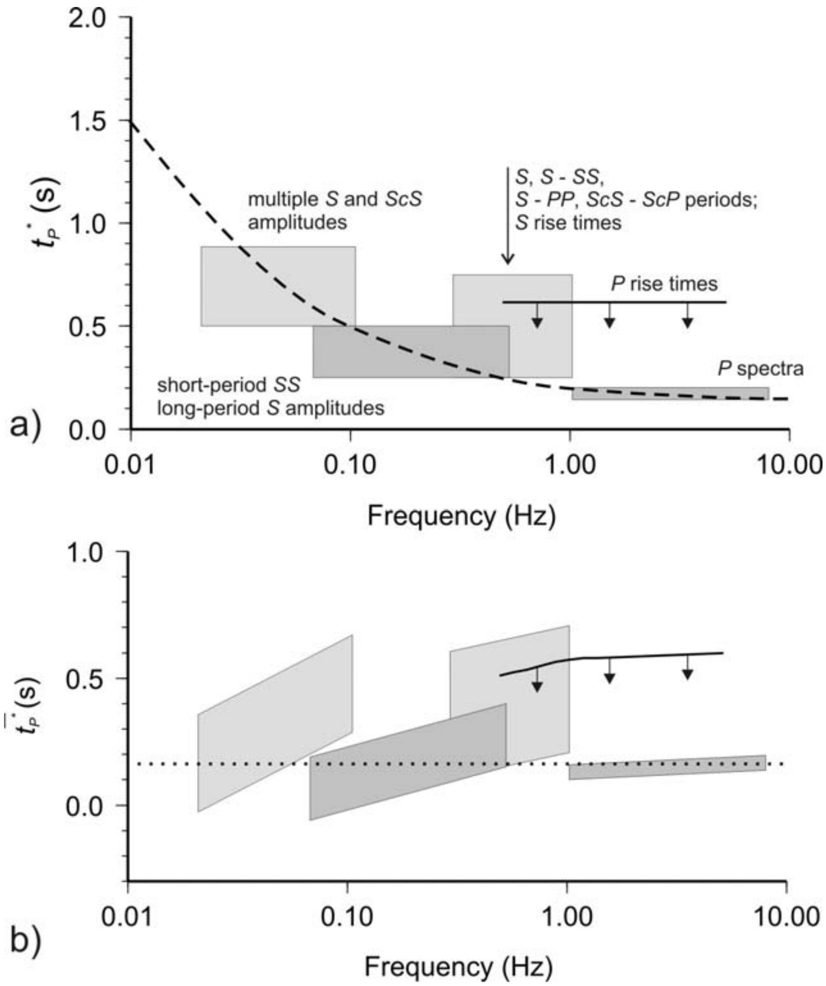

Fig. 2. GS-independent t^* interpretation technique: a) the same data as in Fig. 1a with $t_P^*(f)$ trend estimated from the data by eye (dashed line); b) the trend removed by using eq. (9) and producing a GS-independent $\bar{t}^*$. Note that after this correction, the attenuation can be considered as frequency-independent, with $\bar{t}^* \approx 0.18\,\mathrm{s}$ (dotted line in plot b)).

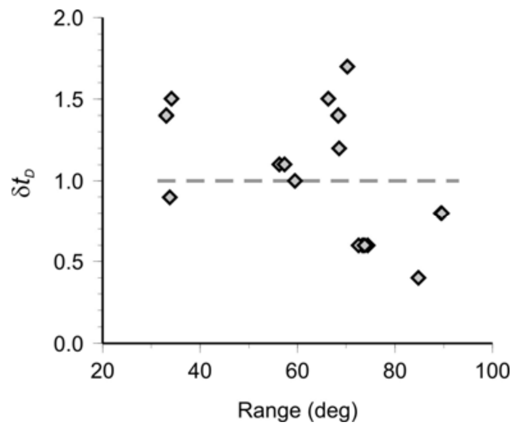

Fig. 3. Long-period body-wave dispersion measurements from Doornbos (1983). Range-independent δt_D = 1 s is indicated by a dashed line.

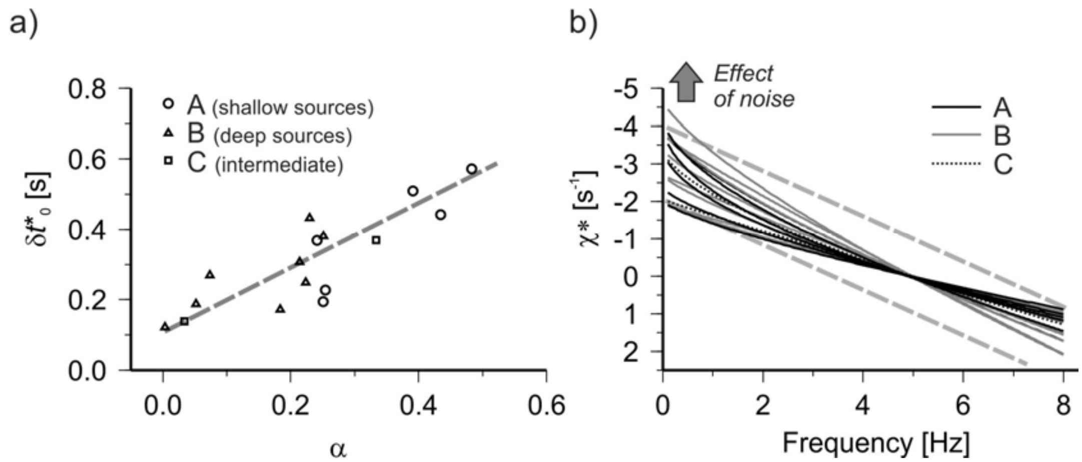

Fig. 4. Summary of frequency-dependent upper-mantle *Q* observations at 0.08–8 Hz by Shito *et al*. (2004): a) α and δt_0^* results for three groups of earthquakes. Note the trend of α and δt_0^* increasing with source depths (dashed line). b) Attenuation-coefficient variations for spectral ratios, reconstructed for the same groups from ($\alpha, \delta t_0^*$) pairs in plot (a). Arrow indicates the effect of noise increasing the spectral ratios at low frequencies. Dashed grey lines schematically indicate one-standard deviation error bounds from a linear trend on the spectral-ratio stacks.

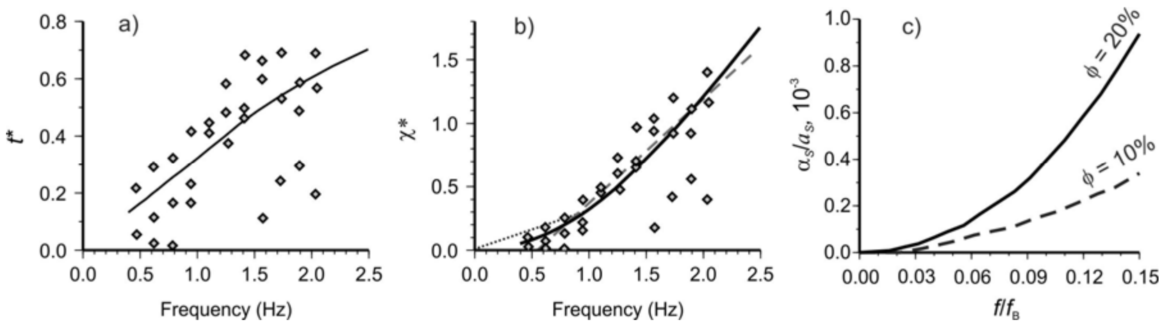

Fig. 5. Inner-core attenuation data between 147° and 151° ranges derived from *PKIKP* to PKP_{BC} spectral ratios by Doornbos (1983): a) the original t^* data plotted in linear frequency scale. Line indicates the interpreted t^* trend; b) the same data in χ^* form; c) comparison to *S*-wave attenuation model in saturated porous sandstone for two porosities, as labelled. (Bourbié *et al*., 1987; Morozov, 2010d). Attenuation coefficients α_S are normalized by $a_S = 2\pi f_B/V_S^0$, where V_S^0 is the *S*-wave velocity at frequency $f \to 0$, and f_B is Biot's (1956) characteristic frequency. In plot b), the slope of the dotted line indicates the traditional value of $t^* \approx 0.3$ s at 1 Hz, and dashed line shows a linear-$\chi^*(f)$ interpretation.

Appendix A. Phase-velocity dispersion

Body-wave dispersion relations are often derived from Kramers-Krönig integrals and relate the frequency-dependence of phase velocity to Q. In this Appendix, we show that the meaning of this Q corresponds to the "effective" Q_e given in eq. (3).

Following Aki and Richards (2002, p.167–169). consider a harmonic wave of frequency ω, traveling in a uniform medium with phase velocity $V(\omega) = \omega/k$, with spectral amplitude $u(\omega) = \exp(-i\omega x/V_\infty + ikx - \alpha x)$, where $k(\omega)$ is the wavenumber, $\alpha(\omega)$ is the attenuation coefficient, and $V_\infty = V(\omega)|_{\omega\to\infty}$. This expression describes a harmonic wave with an infinite-frequency onset at point x occurring at time $t = 0$. To ensure that also $u(t) \equiv 0$ for all $t < 0$ (causality), the Kramers-Krönig identities require that the imaginary part of the wavenumber (α) is uniquely related to its real part (k), and *vice versa*. If we consider limits of $\alpha_0 = \alpha(0)$ and V_∞ to be finite, then $k(\omega)$ turns out to be insensitive to α_0, and $\alpha(\omega)$ is insensitive to V_∞, and, the Kramers-Krönig integrals relate the deviations of $k' = k - \omega/V_\infty$ and $\alpha' = \alpha - \alpha_0$ from these reference levels (*ibid*):

$$\begin{cases} k'(\omega) = \dfrac{\omega}{\pi} P\displaystyle\int_{-\infty}^{\infty} d\omega' \dfrac{\alpha'(\omega')}{\omega'-\omega}, \\ \alpha'(\omega) = -\dfrac{\omega}{\pi} P\displaystyle\int_{-\infty}^{\infty} d\omega' \dfrac{k'(\omega')}{\omega'(\omega'-\omega)}. \end{cases} \tag{A1}$$

Parameters V_∞ and α_0 can also be viewed as regularization constants for the integrals in eqs. (A1), which are otherwise divergent (Nussenzveig, 1972).

Causality relations show that if some wave experiences attenuation ($\alpha > 0$), it must also exhibit phase-velocity dispersion, and *vice versa*. In principle, these equations allow expressing the phase-velocity spectrum if attenuation is known at all frequencies. However, these integrals converge very slowly near $\omega' \to \infty$ and $\omega' \to \omega$, and therefore much of the information required for using these expressions to predict either $k(\omega)$ or $\alpha(\omega)$ lies in the regions of unphysically high or low frequencies.

The case of α proportional to ω is of particular interest, because there exists good evidence for it in both observations (Morozov, 2008, 2010a,b) and theory (Morozov, 2010d). However, in this case, the integral in the first equation in (A1) is divergent and needs to be regularized. Such regularization can be done, for example, by assuming that $\alpha \approx \alpha_0 + \alpha_1\omega$ within the seismic frequency band but flattens out at some high frequencies $|\omega| >> \omega_0$ (modified after Azimi *et al.*, 1968),

$$\alpha(\omega) = \alpha_0 + \frac{\alpha_1\omega}{1 + \omega/\omega_0}. \tag{A2}$$

From (A1), the corresponding phase slowness is only sensitive to α_1 and ω_0:

$$\frac{1}{V(\omega)} - \frac{1}{V_\infty} = C - \frac{2\alpha_1}{\pi\left[1 - \left(\omega/\omega_0\right)^2\right]}\ln\left(\frac{\omega}{\omega_0}\right) \approx C - \frac{2\alpha_1}{\pi}\ln\left(\frac{\omega}{\omega_0}\right), \tag{A3}$$

where C is yet another regularization constant which is formally equal infinity in order to satisfy a finite value of $\lim_{\omega\to\infty} V(\omega) = V_\infty$. We can remove this constant by switching the reference velocity from V_∞ to $V_0 = V(\omega_0)$:

$$\frac{1}{V(\omega)} - \frac{1}{V_0} \approx -\frac{2\alpha_1}{\pi}\ln\left(\frac{\omega}{\omega_0}\right). \tag{A4}$$

Denoting, in accordance with our definition of Q_e, $2\alpha_1 = Q_e^{-1}/V_0$, the velocity dispersion law becomes:

$$V(\omega) \approx \frac{V_0}{1 - \frac{1}{\pi Q_e}\ln\left(\frac{\omega}{\omega_0}\right)}. \tag{A5}$$

Equation (A5) shows the general logarithmic phase-velocity increase with frequency in the presence of attenuation, which is supported by many attenuation models (*e.g.*, Carcione, 2007). Parameters ω_0 and V_0 in (A5) represent arbitrary constants on which the resulting values of phase velocities may depend very strongly. However, for Q and ω_0 satisfying $\pi Q_e >> \left|\ln\left(\omega/\omega_0\right)\right|$ and consequently $V \approx V_0$ within the observation frequency band, the ratios of $V(\omega)$ taken at frequencies ω_1 and ω_2 no longer depend on these regularization parameters (Aki and Richards, 2002, p. 170):

$$\frac{V(\omega_1)}{V(\omega_2)} \approx 1 + \frac{1}{\pi Q_e}\ln\left(\frac{\omega_1}{\omega_2}\right). \tag{A6}$$

This ratio is often transformed into the "physical dispersion" relation and attributed to the viscoelastic moduli (Dahlen and Tromp, 1998; p. 218), for example:

$$\frac{\mu(\omega_1)}{\mu(\omega_2)} \approx 1 + \frac{2}{\pi Q_\mu} \ln\left(\frac{\omega_1}{\omega_2}\right) . \tag{A7}$$

The above equations can also be expressed as variations of V and μ with frequency:

$$\delta(\ln V) \approx \frac{1}{\pi Q} \delta(\ln \omega) \text{ , and } \delta(\ln \mu) \approx \frac{2}{\pi Q_\mu} \delta(\ln \omega) . \tag{A8}$$

In order to understand the meaning of Q in dispersion laws (A8), it is useful to try adding another "relaxation mechanism" with a different ω_0 to eq. (A2):

$$\alpha(\omega) = \alpha_0 + \frac{\alpha_1 \omega}{1 + \omega/\omega_{0,1}} + \frac{\alpha_2 \omega}{1 + \omega/\omega_{0,2}} . \tag{A9}$$

By combining such terms, practically any monotonic $\alpha(\omega)$ functions increasing not faster than ω can be constructed. It is easy to verify that subject to the same approximation $V \approx V_0$, expression (A6) becomes:

$$\frac{V(\omega_1)}{V(\omega_2)} \approx 1 + \frac{1}{\pi}\left(\frac{1}{Q_{e,1}} + \frac{1}{Q_{e,2}}\right) \ln\left(\frac{\omega_1}{\omega_2}\right) . \tag{A10}$$

where $1/Q_{e,1,2} = 2V_\infty \alpha_{1,2}$. This shows that velocity dispersion is sensitive to the derivative $d\alpha/d\omega = \alpha_1 + \alpha_2$ near $\omega = 0$ rather than to the cut-off frequency ω_0. Therefore, the Q-factor in expression (A5) and (A6) should correspond to the derivative $d\chi/d\omega$ evaluated after the zero-frequency limit of χ is removed, which corresponds to our Q_e value (eq. (3)). This is reflected in our notation in (A5) and (A6).

In summary, causality constraints require velocity dispersion in the presence of attenuation, and yet the exact form of this dispersion should be determined from the specific wave models. Normally, any mechanical system possessing a time-domain (Lagrangian) description should behave causally. Similarly, Kramers-Krőnig integrals require a frequency-dependent attenuation $\alpha(\omega)$ at least at the very high and very low frequencies. However, for practical purposes, this requirement is not very useful, because the low-frequency cut-off below which Q^{-1} must decrease is about 10^{-99} Hz for $Q \geq 30$ (Futterman, 1962). Thus, the causality principle only weakly constrains the properties of the medium and does not constrain any definite frequency dependence of V or Q within the seismological frequency band.