Lecture 1: Introduction

- Problems and objectives
- Scales: observational and of microstructure,
  - Importance of the macroscopic scale, homogenization
- Types of anelasic phenomena: transient deformations, oscillations, waves
- Key concepts: characteristic times, empirical modulus, Q-factor, attenuation, dispersion
- Two approaches to anelasticity:
  - Conventional: the viscoelastic model
  - Approach of this course: macroscopic (Lagrangian) continuum mechanics

---

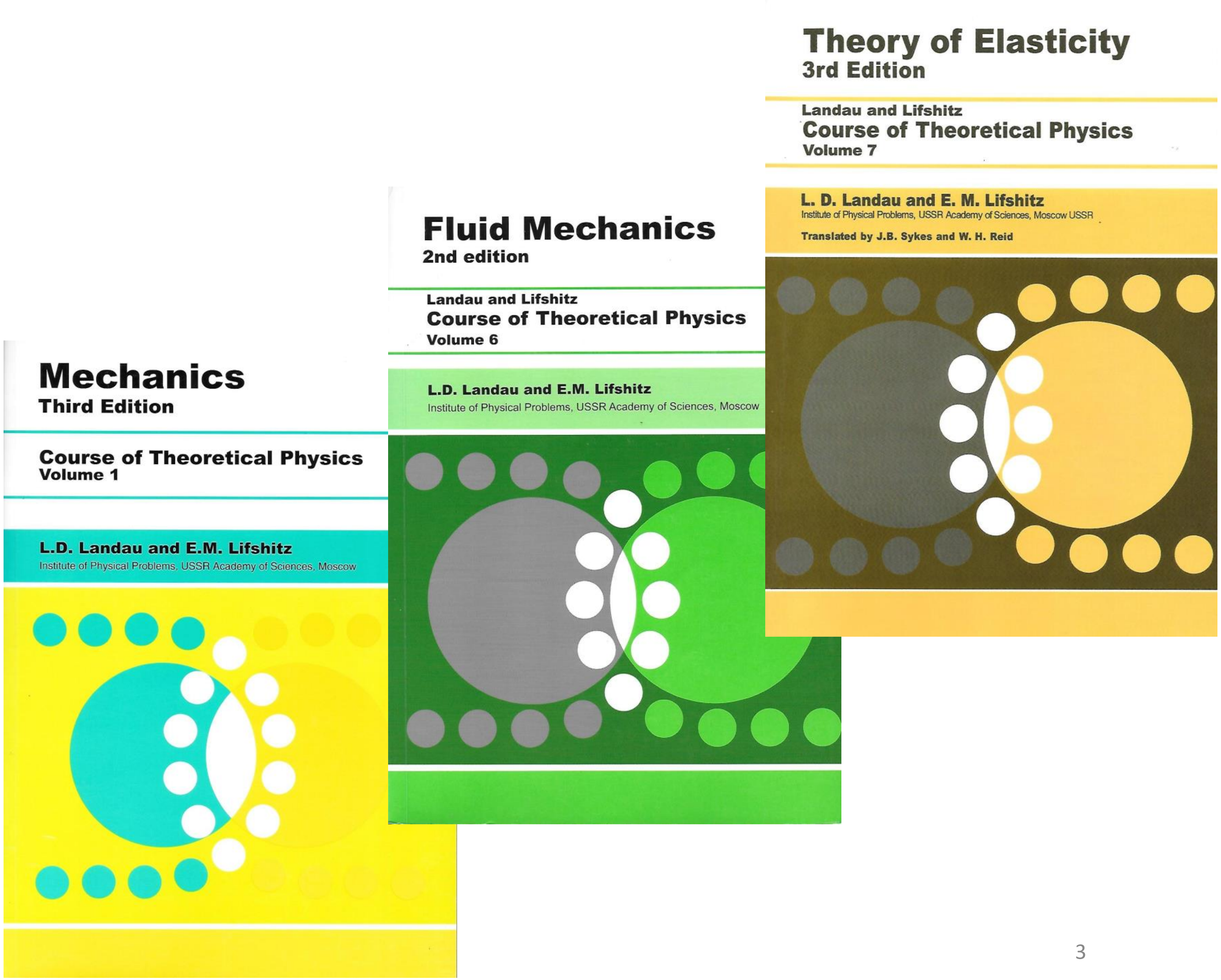
- Reading: Chapter 1 in the text
- Website: http://seisweb.usask.ca/classes/UPC-2024/WWW/index.html

Objectives: “How Rock Deformation Works”

- The goal is to describe phenomena occurring within rock during deformations
  - Passage of a seismic wave, laboratory experiment (weak)
  - Pore-fluid flow
  - Tectonic or geotechnical impact on the reservoir (strong)
  - Deformations with after-effect (continuing after the impact)
- Explaining the data in any of these experiments empirically is usually not that difficult
  - Creep laws
  - Viscoelastic laws
  - Darcy law, pipe flows, etc.
- The problem is:
  - To describe all of the above phenomena consistently
  - To identify **rock properties** which **can and should be measured**
  - To find the driving mechanisms, not just empirical relations
- **Knowledge of physics helps achieving these goals**

General approach

- Methods of theoretical physics are well established
- They allow rigorously and completely solving all problems in rock physics and seismic waves
- In this course, I illustrate some applications of the general principles of Lagrangian mechanics and thermodynamics to these problems



Types of deformations and terminology

- Four deformation types are differentiated for rock: Elastic, inelastic, anelastic, plastic
- These types are differentiated by studying the behavior of rock in several experiments
- Usually, we need to pay attention to time dependencies of strain or stress, and dissipation of heat

Deformation type	**Viscoelastic**		
	Elastic	**Inelastic**	
Thermodynamic type	Reversible[1] (no dissipation of heat)	Irreversible [1] (heat dissipates)	
Time dependence of strain [2]	Synchronous with stress	Delayed relative to stress[3]	
Deformation sub-type		**Anelastic**	**Plastic**
Recoverability by unloading [2]	Recoverable	Recoverable	Not recoverable

[1]) In a loop of loading/unloading.

[2]) In a step-function stress experiment.

[3]) Also postulated time dependence of strain-stress relations in viscoelastic model.

Elastic, anelastic, and plastic deformations

- Described by empirical behavior under load:
- Burgers' model contains examples of all three of the basic deformation types
  - Elastic – the body instantly returns to the original state when load is removed
  - Anelastic – the body restores its state after some time delay
  - Plastic – the deformation can extend indefinitely
- Note that different deformations are associated with some "internal variables":
  - Movements of groups of grains
  - Pore fluid flows
  - Heat flows, etc.

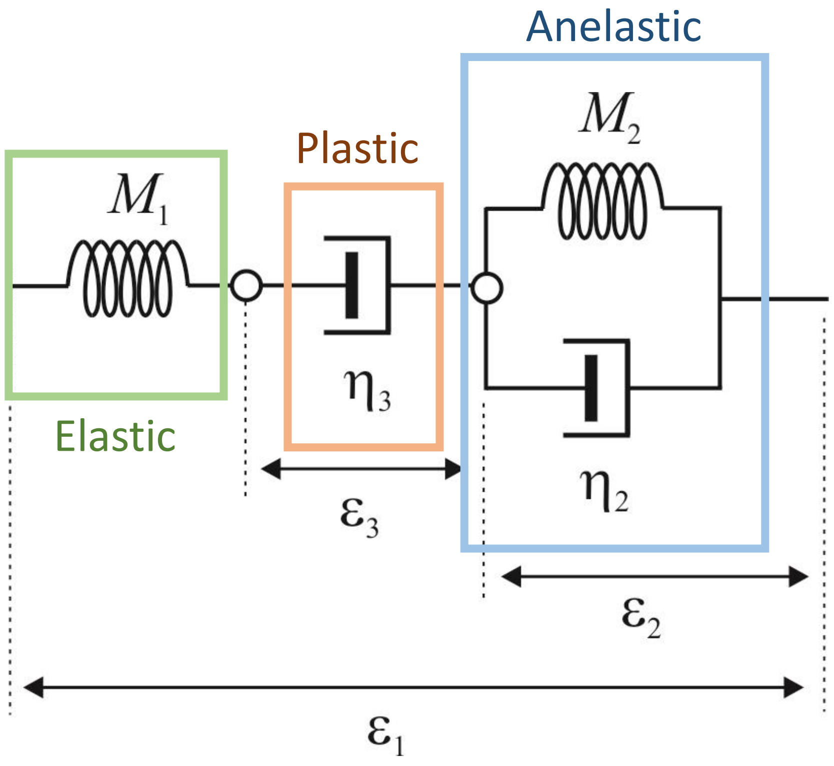
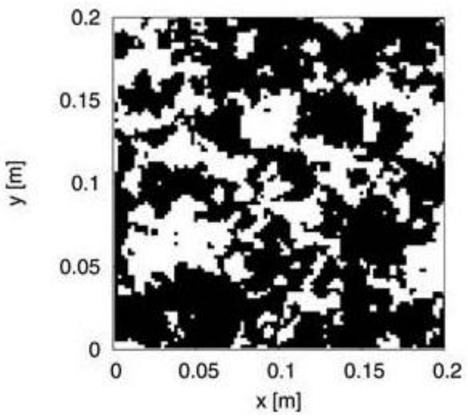


Figure 1.1.
Burgers' model of deformation. Stress σ is applied to the ends of this chaoin of mechanical elements and causes observed deformation ε_1. This deformation includes an anelastic part (involving both elasticity and viscosity, ε_2), plastic part (viscosity only, ε_3), and pure elastic part (the unlabeled deformation of the spring M_1). **Note that anelastic deformations always include movements of some internal degrees of freedom** (white dots in this plot) which are not observed in the mechanical strain-stress testing experiment.

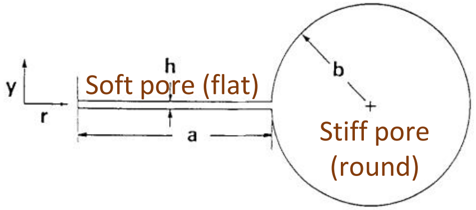
Porous rock

Popular models of pore space

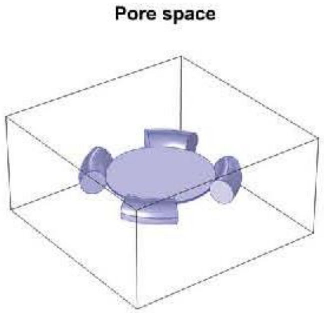
- Most work in exploration rock physics focuses on porous rock
- Gassmann's model is the simplest representation of porous rock:
  - Fluid-filled pores are interconnected so that they have a common pressure
  - Pore infills have zero shear rigidity
  - Relatively slow deformations are considered: rock frame and fluid are at quasi-static equilibrium
- Physically, this means a description of rock by only three macroscopic parameters:
  - Two dilatations: of the whole rock and pore fluid
  - One shear of the whole rock
  - Gassmann's equation (relation between four observed moduli K_U, K_D, K_s, and K_f) is an automatic consequence of having only three independent parameters
- The microstructure is completely unimportant when considering static deformations
  - All models should satisfy Gassmann's equation for near-static deformations



Fractal structure with patchy saturation (Rubino and Holliger , 2013)



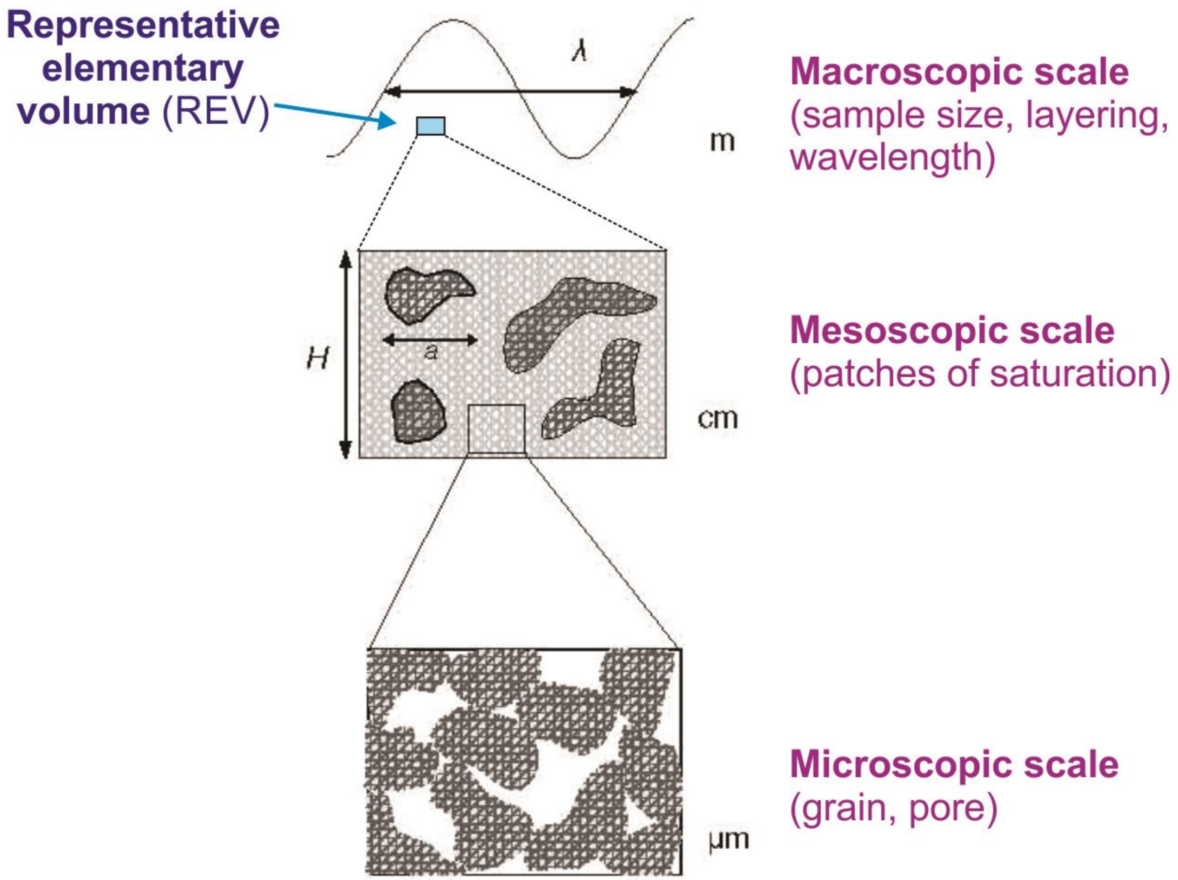
Microcrack with "squirt flow" (Murphy et al., 1986)



Numerical model of connected pores (Alkhimenkov and Quintal, 2022)

Scales

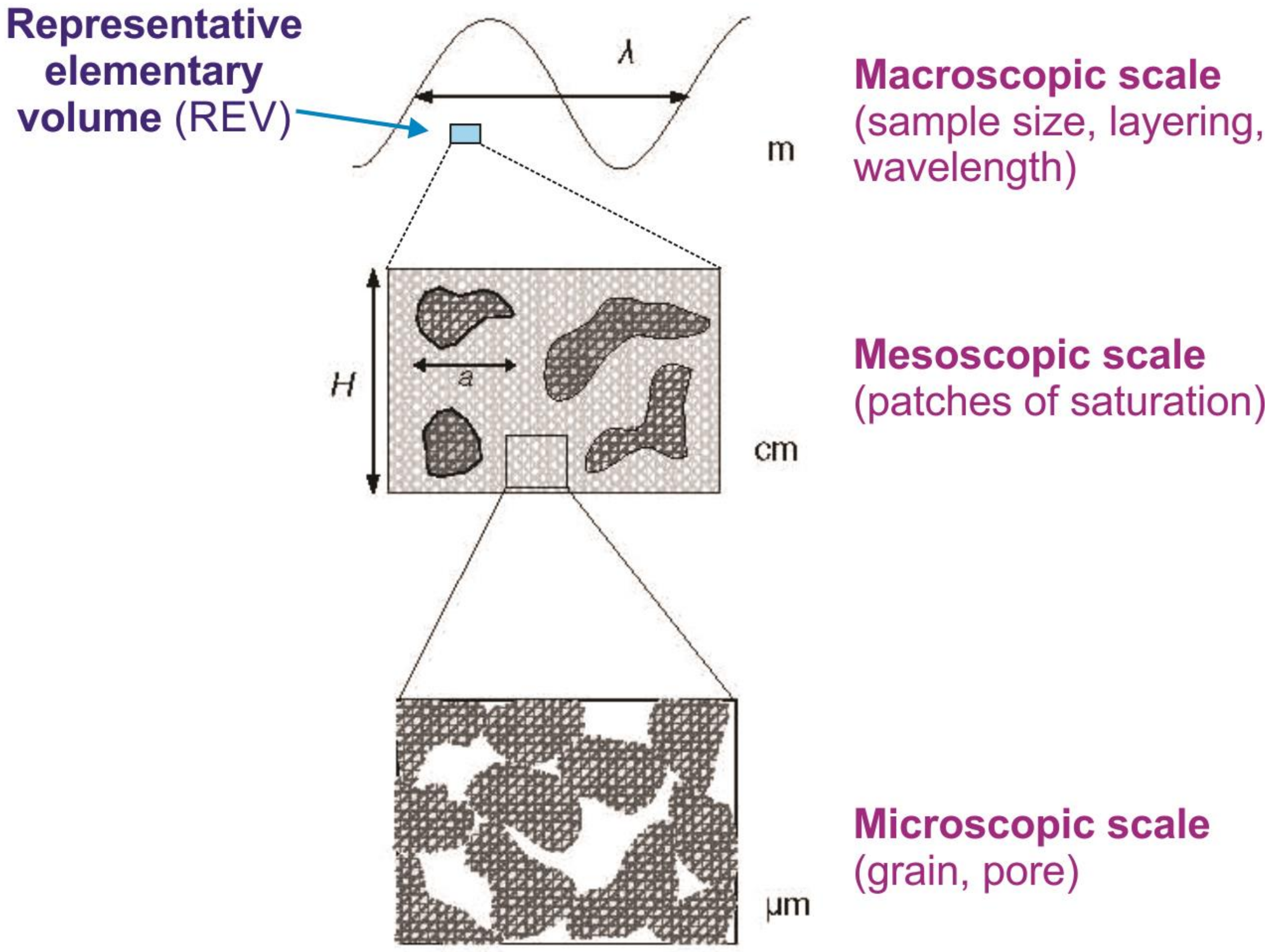
- When considering non-static deformations like waves, more detail of microstructure is revealed. This detail is often characterized by the notion of spatial scales
- Four characteristic spatial scales of investigation should be recognized:
  - Microscopic, mesoscopic, and macroscopic scales of rock structure
    - These scales correspond to characteristic times (or frequencies) of relaxation processes
  - Observational scale at which the equations of deformation and motion are written
    - This scale is given by the Representative Elementary Volume (REV) scale
- At smaller scales, large numbers of parameters are needed to describe rock, for example:
    - Sizes or shapes of pores, gas bubbles, patches of saturation
    - Radii of capillary menisci
    - Dimensions and aspect ratios of “squirting” microcracks
- However, this detail is unobservable and unimportant in macroscopic experiments



Observation scale

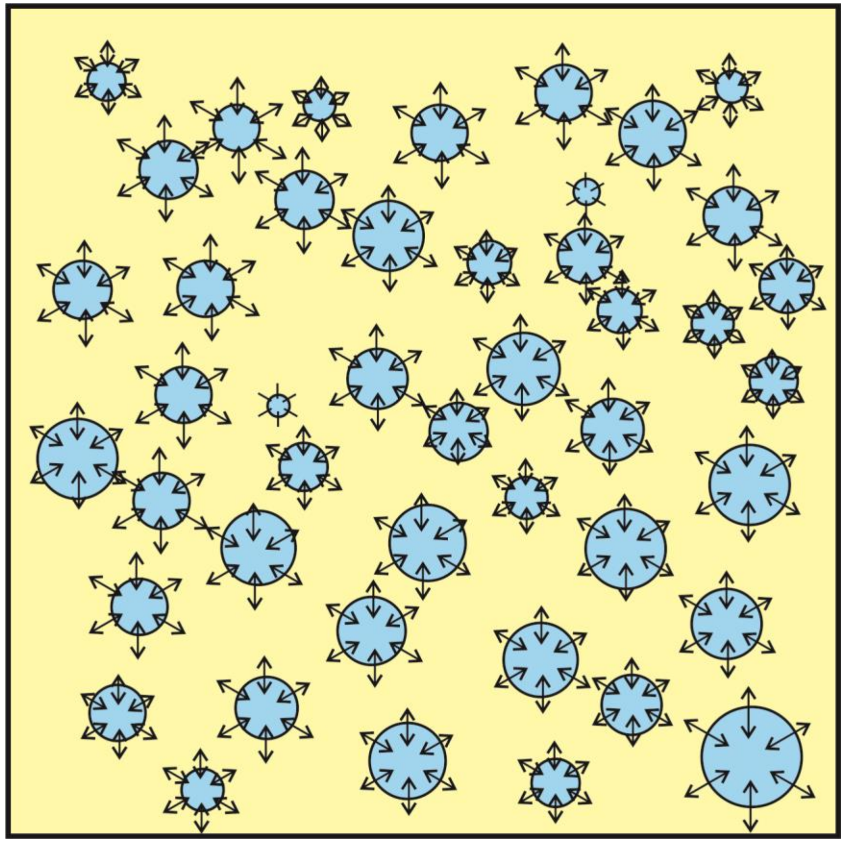
- Importantly, only the macroscopic REV scale is seen in practical observations in laboratory rock physics or in seismic waves

- At this scale, we should only use averaged material properties:
  - Porosity (scalar ϕ and porosity tensor α)
  - Density
  - Effective elastic moduli
  - Permeability
  - Thermal properties (specific heat, heat conductivity)
  - Anisotropy
  - Elastic wave speeds, impedance
  - Other properties to describe anelasticity (which ones? This is the big question!)



Rock-structure scales

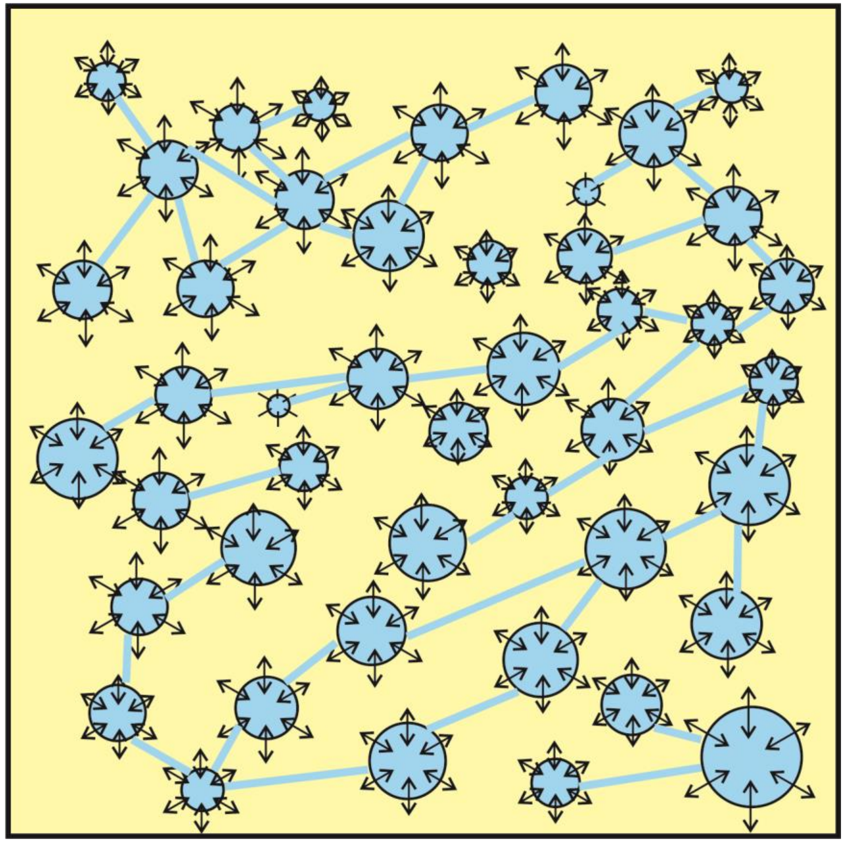
- Here is a model of a rock with spherical pores, subjected to oscillating pressure
- Question 1: What is the scale of this microstructure?
  - Answer: apparently "microscopic" – the size of the pores. However, there are many sizes of pores here (and more in real rock).
- Question 2: What is the scale of this pore-fluid flow?
  - Answer: the same microscopic, or "local"
- Question 3: Is this Gassmann's rock?
  - Answer: In principle yes, but the "average" pore pressure has to be defined in an intricate way
  - In Gassmann's rock, pores should be identical or connected, so that they have the same pressure. In this model, smaller pores have larger pressures
- Question 4: Is this Biot's rock? Biot's model is known as an example of "global" scale pore-fluid flow.
  - Answer: No. Biot's rock requires permeability of the whole REV



Note that this difference is a question of pore connectivity but not of scale

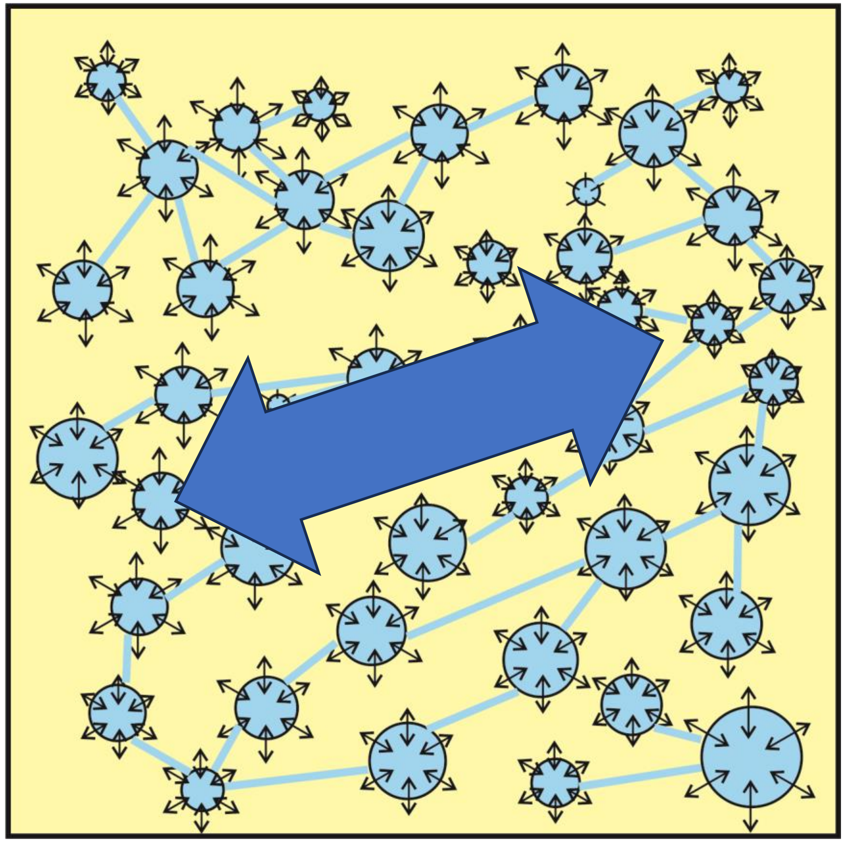
Pore-flow scales

- Here is how the Biot's model look like:
- Note that the difference is not in pore shapes or characteristic flow scales but in pore connectivity
  - The connectivity equalizes the pore pressure, as required by Biot's model
- Thus, the difference is really in the permeability of the rock
- The pore-fluid flow is still principally "local" (black arrows; expanding pores of various sizes)
  - Micro- and mesoscopic
  - The difference in scales is not that important because it is complicated by variations of shapes and sizes of pores and other elements of structure
- Question 4: So, where is the "global" (macroscopic) Biot's flow in this picture?



“Global” Biot’s flow

- Answer: There was no global flow in that model
- The “global” flow would be observed in a different experiment:
  - If a macroscopic-scale pressure gradient is imposed in some direction, a net flow is induced through the conduits between pores
  - This results in a pore flow in the same direction (if the random microstructure is isotropic)



Deformation patterns, variables, and material properties

- In reality, the difference between different deformation phenomena is not in their apparent scales but in different mechanical properties related to certain shapes of deformation
- On the observational scale, these different patterns are described by different types of macroscopic variables:
  - Deterministic flows are measured by net (average, observable) displacement vectors:

$$\bar{\mathbf{u}} \approx \mathbf{u}_{\text{net}}$$

  - For random divergent movements, there is no net displacement, and the flows need to be measured by average strains (divergence or shear). These are basically scalar quantities:

$$\bar{\varepsilon} \approx \varepsilon_{\text{net}} + \bar{\varepsilon}_{\text{div}}$$

- The big question is what mechanical parameters can be selected to characterize the medium, and not just to describe the resulting flows
- These parameters are not spatial scales of the flows but macroscopic mechanical properties. These material properties can be of only three kinds:
  - Elastic moduli for each of the above forms of deformation
  - Viscosities associated with these deformations
  - Inertia of the flows

The question is how to identify these material properties.
This is what we try doing in this course

Deformations

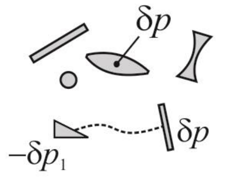
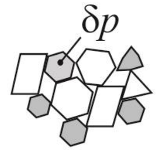
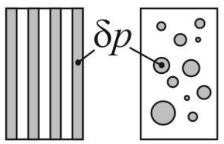
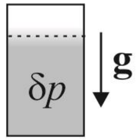
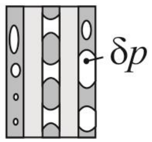
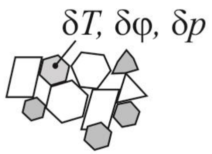
- Deformation of a material involves numerous microscopic-scale physical phenomena
- By their macroscopic (observable) effects, they can be classified into four groups:
  - Viscosity-type (mechanical friction)
  - Thermal (involving temperature variations)
  - Kinetic (like chemical interactions, diffusion)
  - Scattering (“elastic”, without mechanical energy loss)

Mechanism	**Viscosity** (due to strain rates)	**Thermal** (due to thermal flows)	**Kinetic** (due to internal variables)	**Scattering** (due to elastic effects)
1) Atomic and molecular processes:	✓			
a) Relaxation by motion of solute atoms, in metals or on strain gradient;			✓	
b) Relaxation by dislocation motion (amplitude or frequency dependent); a special case is the “high-temperature background” (increase of Q with frequency);			✓	
c) Relaxation by molecular rearrangement in polymers;	✓		✓	
d) Relaxation by atom pair reorientation;			✓	
e) Relaxation by diffusion of atoms (at high temperature in metals);	✓		✓	
f) Relaxation by phase transitions;			✓	
g) Relaxation by electron viscosity;	✓		✓	
h) Relaxation by point-defect motion			✓	
2) Coupled-field effects:				
i) Thermoelastic dissipation*;		✓		
j) Fluid flow effects in porous materials containing fluids;	✓			
k) Piezoelectric and magnetoelastic effects;	✓			
l) Phonon-phonon scattering;*	✓			✓
m) Electron-phonon scattering.				✓
3) Effects due to heterogeneity:				
n) Relaxation in multiphase structured materials;				✓
o) Grain boundary slip in polyscrystalline materials, such as metals, ceramics, or rocks;	✓			
p) Filtration fluid flows in porous materials.	✓			
q) Movement of dislocations in crystalline lattices;	✓			
r) Elastic scattering (reflections and wave mode conversions) on heterogeneities and boundaries of the body.				✓

*) Fundamental, inherent to all materials.

Wave-Induced Fluid Flow (WIFF)

- One specific “coupled-field” mechanism (red box ‘j’ in the preceding slide) is particularly strong in porous rock containing fluids (brine, gas, oil)
- This mechanism consists in interaction of porous solid frame with a relatively mobile fluid
    - It produces the so-called “Wave-Induced Fluid Flows” (WIFF)
- WIFFs can take on numerous forms and occurs on all three spatial scales (figure on the right)
    - However, on the REV scale, the averaged effects of all these WIFFs are similar
    - The different types of WIFF are differentiated by the characteristic times of relaxation processes

a		Type I WIFF: trapped pore fluids, micropores of various shapes, communicating micropores, squirt flows, multiple porosities	Microscopic
b		Grains, inclusions, ice, bitumen	Mesoscopic
c		Type II WIFF: patchy saturation, thin layers	Mesoscopic
d		Laboratory-scale WIFF Gravity-driven saturation layering	Macroscopic
e		Capillary effects, gas bubbles	Microscopic, Mesoscopic
f		Thermoelastic, piezoelectric effects	Microscopic, Mesoscopic

Characteristic times and frequencies

- Different anelastic mechanisms (WIFF or other) are recognized by measuring the characteristic relaxation times τ_c associated with them
- The times are measured by methods discussed a little later. The frequency-domain method is shown on the right

$$\tau_c = \frac{1}{2\pi f_c}$$

- Note that different mechanisms lead to different dependencies of the attenuation coefficient $\chi(f)$ and inverse Q-factor $Q^{-1}(f)$ on frequency f

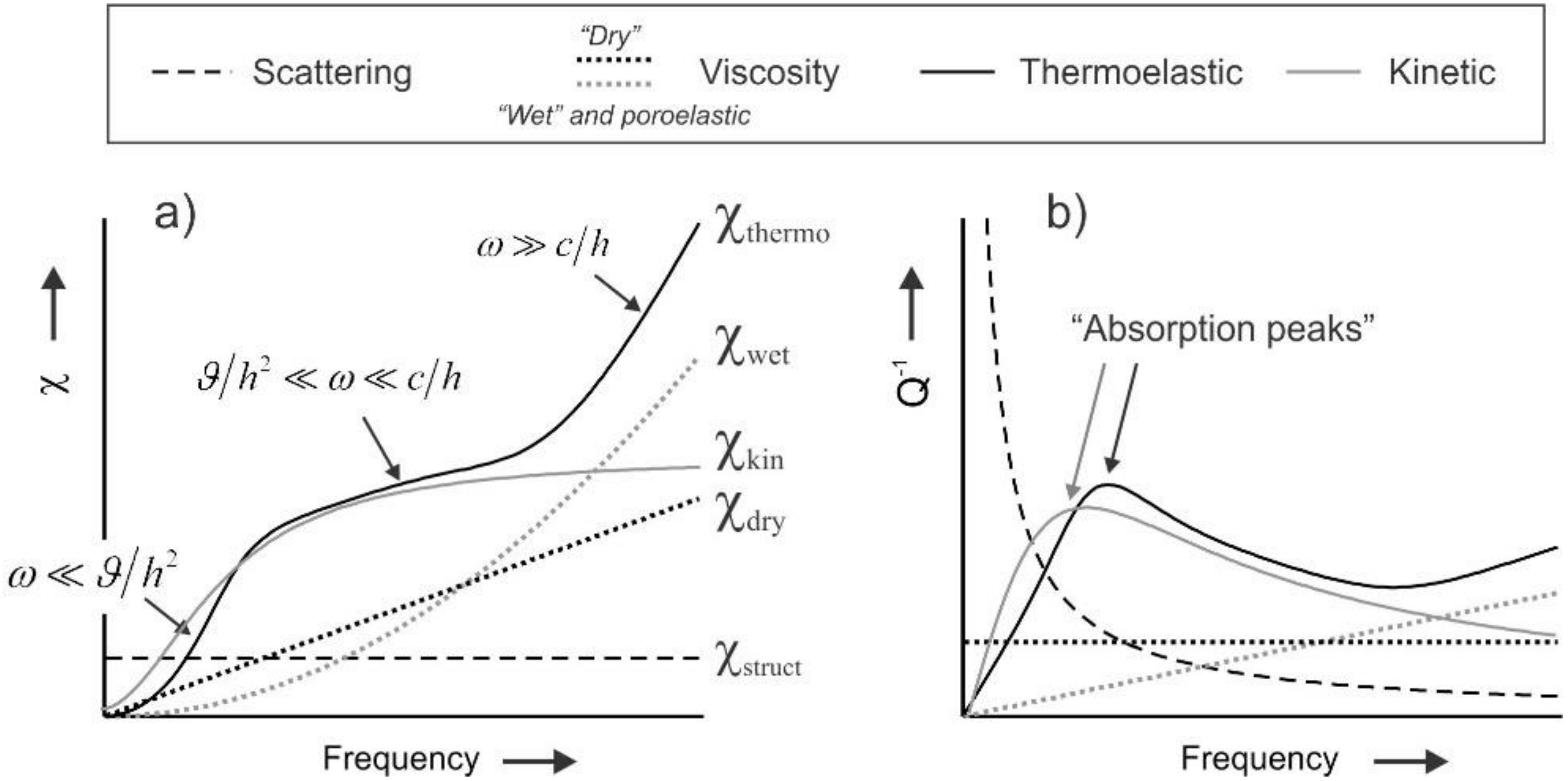


Figure 1.4
Schematic frequency dependencies of wave energy dissipation for different mechanisms of anelasticity in a material without internal variables: a) temporal attenuation coefficient χ chapter 5), b) inverse quality factor Q^{-1} . Scaling of the axes is arbitrary.

For viscosity, two functions are shown, corresponding to linear 'wet', gray dotted line), and nonlinear ('dry', black dotted line). The "wet" viscosity case also represents the saturated porous rock. Expressions in the labels in (a) indicate three different regimes of thermoelastic dissipation (chapter 6).

Relaxation time and rock viscosity

- When stress is applied or removed, rock bodies often exhibit time-dependent responses with different relaxation times τ
  - Currently, it is believed that the different times are related to different **spatial scales** of pore-fluid flows (WIFF)
  - For example, relaxation of large patches of saturation occurs slower than relaxation of small pores
- The time τ gives roughly the size of the part of the body (heterogeneity) responsible for this oscillation:

$$l \approx Fc\tau$$

(*c* is the wave speed, and F is some formfactor related to the assumed average shape of the pores)

- This is how numerous models of WIFF are constructed ("squirt flow", patchy saturation, etc.)
  - They basically consist of derivations of factor *F* for some assumed shapes of the pore (sphere, "penny", "wedge", "digital rock" structure, etc.)
  - However, both l and F are practically impossible to determine for real rock structure
- By contrast, in a general and assumption-independent way, the relaxation time simply indicates the characteristic viscosity of rock (without assumptions and approximations):

$$\eta \approx M\tau_r$$

(M is the elastic modulus)

Observations of elasticity and anelasticity

- Having considered the structure and properties of materials, let us now consider observations which help us recognize these properties

- There are three groups of commonly used observations of rock anelasticity:
  1) Time-domain measurements in the lab: Transient (time-dependent) deformations
  2) Frequency-domain in the lab: Deformations of rock samples by sinusoidal (harmonic) oscillating forces
  3) Frequency-domain in the field: Frequency-dependent observations with seismic waves
- We will consider each of these groups separately

Observation 1: Transient deformations

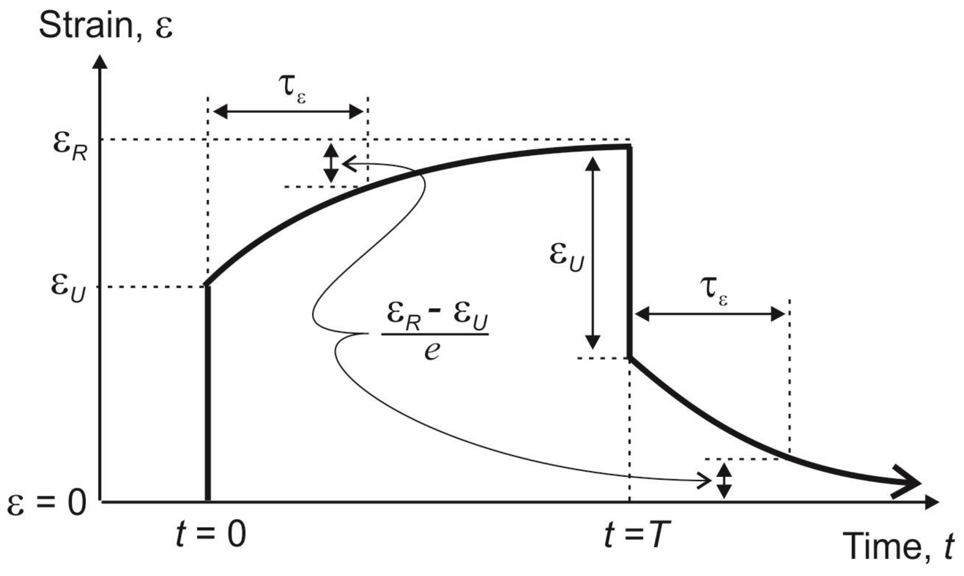


Figure 1.5
Linear relaxation of strain in a rock sample. After a step in stress σ is applied at time $t = 0$, the initial strain becomes ε_U ('unrelaxed'), and it gradually approaches the 'relaxed' strain at $t \to \infty$. After the loading stress is removed at time $t = T$, the relaxation process is repeated in the opposite direction.

Note that with periodic loading and unloading, the pattern of strain lags behind the stress. This is an indication of the stress-strain phase delay for periodic processes.

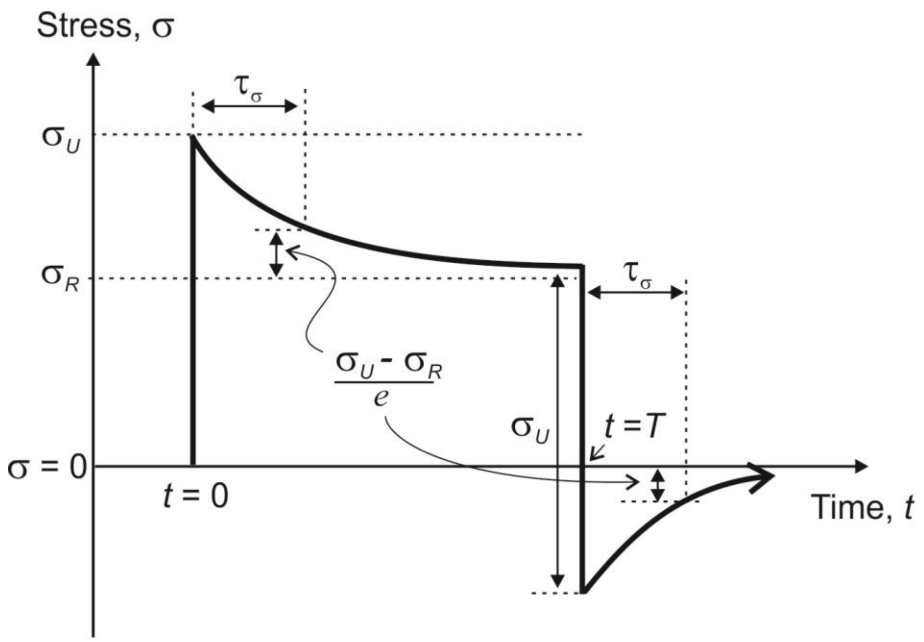


Figure 1.6
Linear relaxation of stress in a rock sample. After a step in strain ε is applied at time t = 0, the initial stress 'overshoots' the equilibrium value and becomes σ_U ('unrelaxed'). With time increasing, the stress gradually reduces to the 'relaxed' (equilibrium) stress at $t \to \infty$. After the strain is removed at time $t = T$, the stress relaxation process is repeated in the opposite direction.

Observation 2: Forced harmonic oscillations

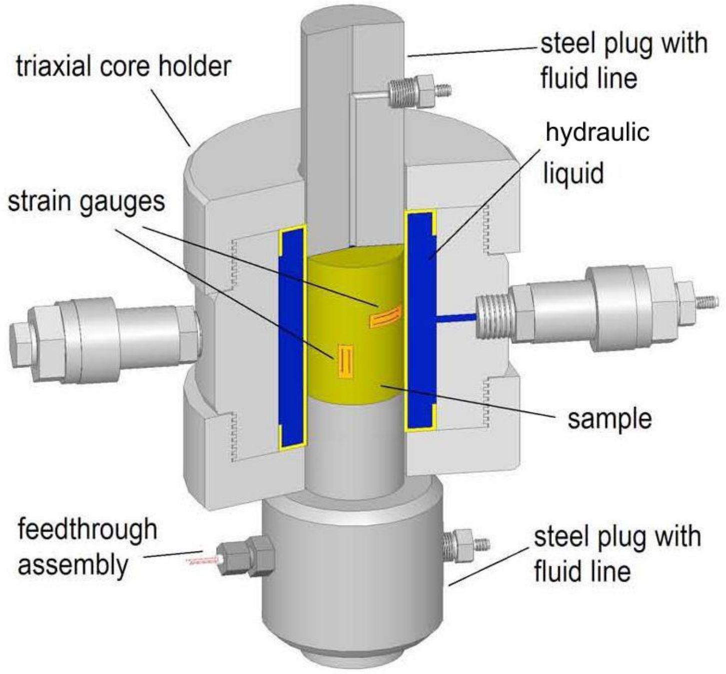
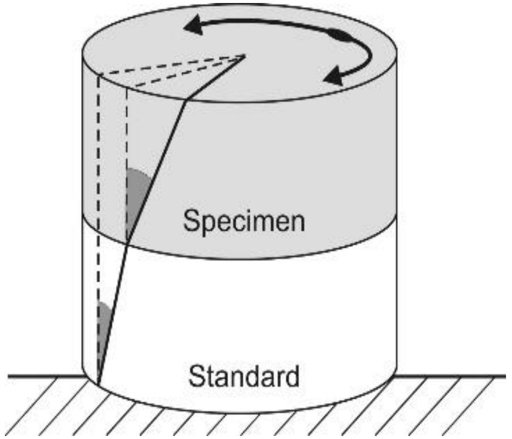


Figure 1.7
An apparatus for Young's modulus measurements (from Mikhaltsevitch et al., 2015). A jacketed cylindrical rock core (yellow) is placed in a column with an aluminum cylinder (standard), so that they receive a common vertical pressure oscillating in time. The resulting axial and transverse strains are measured by a strain gauges attached to the sample (yellow) and also to the standard. Confining pressure (may also be oscillating) is applied using hydraulic fluid surrounding the sample (blue). Pore-fluid pressure is regulated via the fluid line(s) connected at the end(s) of the sample.



Principle of torsional phase-lag Q measurements for shear deformation (Jackson and Paterson, 1993). From the two shaded angles, the ratio of strains in the sample and the aluminum standard element is determined. Because the torque within the specimen and standard is the same, the shear strain of the standard and its shear modulus can be used to determine the stress/strain ratio within the specimen.

Forced harmonic oscillations

- Data (ratio of the strains of the sample and aluminum standard)are presented as "effective modulus" $M(f)$
  - Young's or shear moduli in the preceding slide
- This ratio is a complex value:

$$M \overset{\text{def}}{=} \frac{\sigma^*}{\varepsilon^*} = M'\left(1 - iQ^{-1}\right)$$

- Real part M' is the 'empirical modulus' and argument is the strain-stress phase lag δ
- Arctangent of this phase lag is called the inverse Q-factor:

$$Q^{-1} \overset{\text{def}}{=} -\frac{\operatorname{Im} M}{\operatorname{Re} M} = \arctan \delta$$

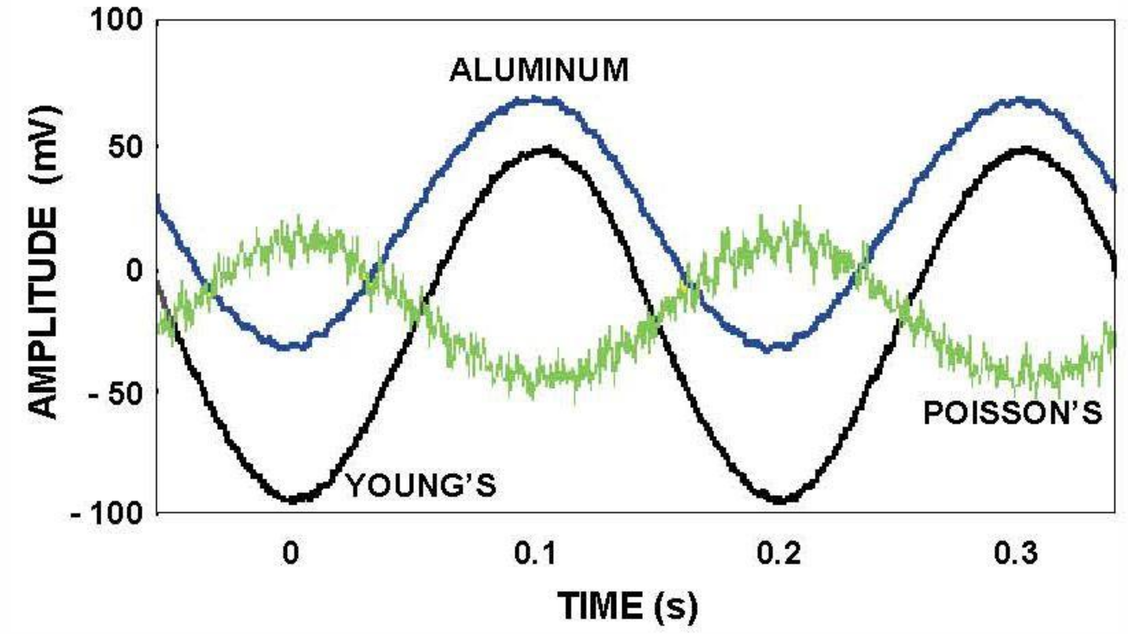


Figure 1.8
Subresonant measurements of Young's modulus (from Batzle et al., 2001) A sinusoidal axial pressure at frequency 5 Hz is applied to a column consisting of an aluminum standard and the rock sample. Vertical strain gauges (Figure 1.7, left) measure Young's modulus deformation (black and blue curves), and horizontal gauges give the Poisson's ratio (green). Note the slight phase lag of the black curve relative to the blue one – this is the measured Q^{-1} (equation on the left).

Dispersion and attenuation of empirical modulus

- The experiment is repeated at multiple frequencies, and observations like shown in this figure are often made:
  - The modulus (real part of the complex modulus $M(f)$) shows an increase at some frequency f_0. This increase is called 'modulus dispersion'
  - Q^{-1} shows a peak at the same frequency. This peak is called 'attenuation' because mechanical energy is dissipated when Q^{-1} is large
- However, note that the peak value of Q^{-1} only depends on the elastic moduli M_R ('relaxed') and M_U ('unrelaxed')
  - This follows from the "Kramers-Krönig relations" (explained later)
- Frequency f_0 is the only true 'attenuation' observation
  - For example, it reduces when more viscous fluid in placed in rock pores

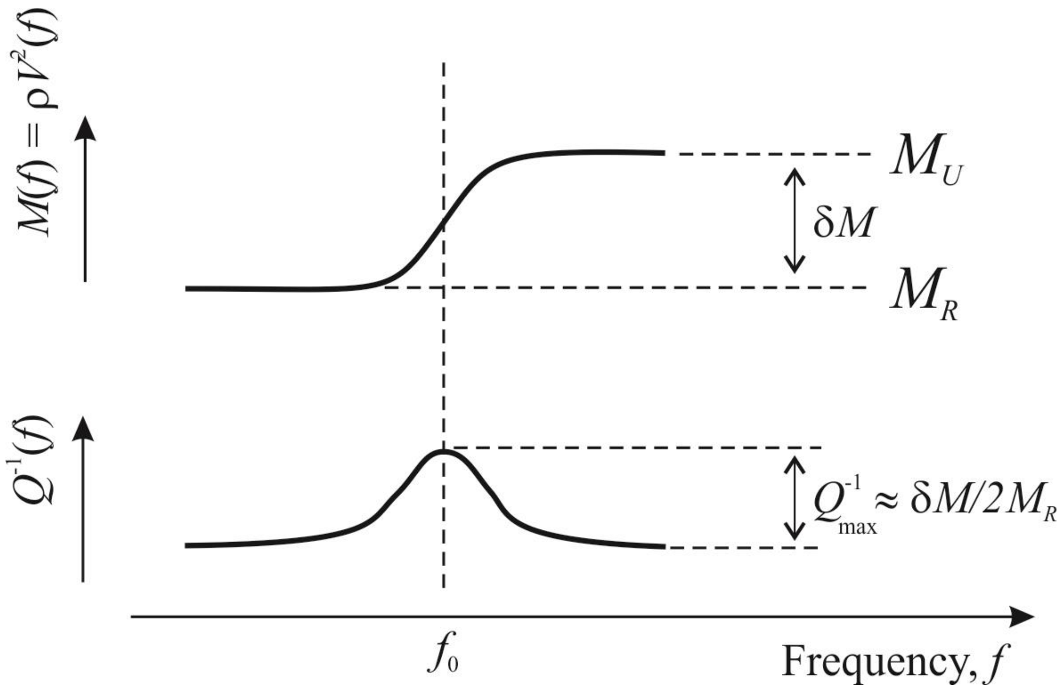


Figure 1.9
Schematic modulus dispersion and attenuation spectra observed in experiments with seismic waves and rock samples. The dynamic modulus M undergoes a step ("dispersion") by some amount of "modulus defect" δM near frequency f0, and the attenuation factor Q^{-1} shows a peak of height proportional to $\delta M/M$ at the same frequency. These particular shapes of the dispersion transition and attenuation peak can be modeled by the standard linear solid (Zener) model (Lecture 2; chapter 5).

Observation 3: Waves

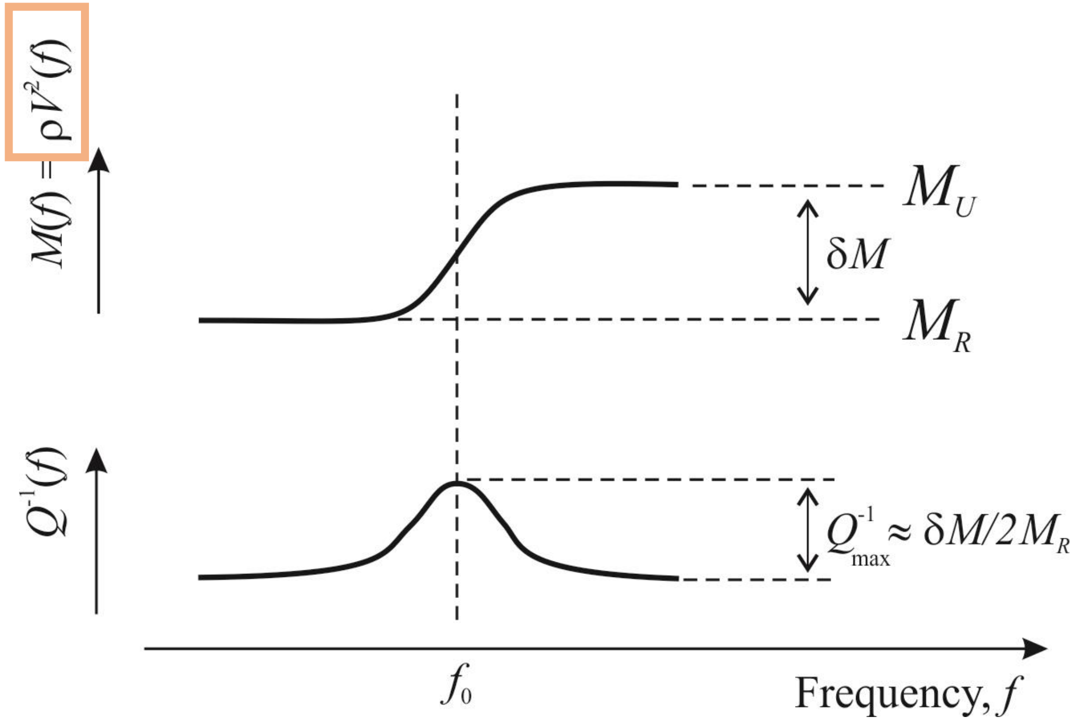
- For a seismic wave, the effective modulus *M* is related to wave velocity c as

$$c = \sqrt{\frac{M}{\rho}} \qquad (\rho \text{ is the density})$$

- Because of the complex-valued $M(f)$, $c(f)$ is also complex-valued and has a similar frequency dependence with a *Q*-factor responsible for imaginary part of slowness:

$$\frac{1}{c} = \sqrt{\frac{\rho}{M}} \approx \frac{1}{c'}\left(1 + \frac{i}{2}Q^{-1}\right) \qquad \text{(slowness)}$$

- What does this Q^{-1} mean for a wave? (see next slide)



Q for waves

- The Q^{-1} is seen somewhat differently when looking at time and distance dependencies of waves
- For time-dependent waves (like free oscillations and seismic coda):

$$A(t) = A_0 \exp\left(-\frac{\omega t}{2} Q^{-1}\right)$$

  - However, A_0 may depend on time too…
- For distance-dependent traveling waves, Q^{-1} describes the decrease of amplitude with distance x:

$$u(x,t) = u_0 \exp\left[-i\omega\left(t - \frac{x}{c'}\right)\right] \exp\left(-\frac{\omega T}{2} Q^{-1}\right)$$

where $T = \frac{x}{c}$ is the travel time

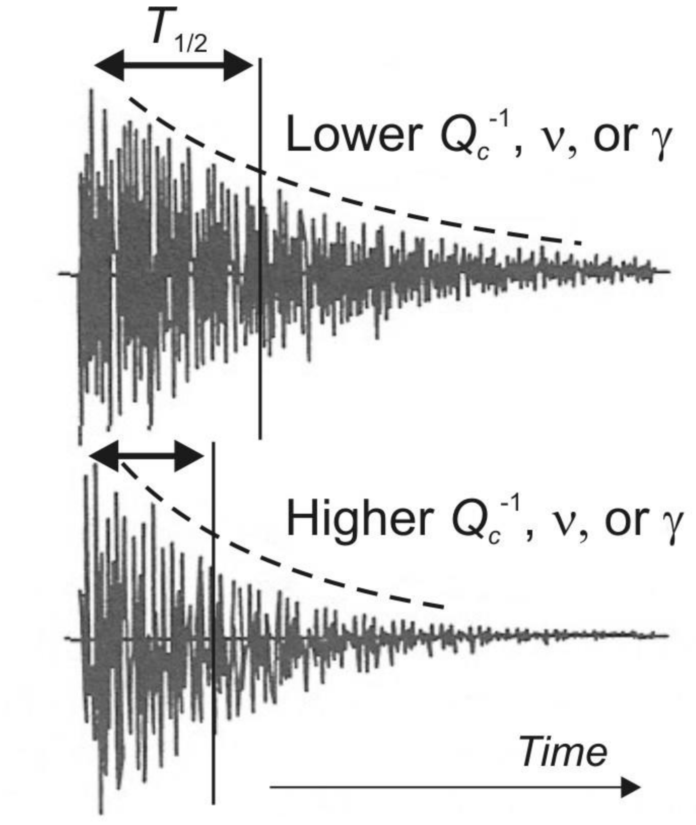


Figure 1.9
Schematic seismograms showing seismic coda waves from an earthquake, with lower and higher attenuation. This is about what you see in an Acoustic Emissions experiment.

Higher apparent attenuation is recognized from shorter duration of the coda $T_{1/2}$, which can be described by larger values of Q^{-1}, ν, or γ (parameters ν and γ describe the geometrical spreading; I do not discuss them in this course).

Theories of relaxation (anelastic) phenomena

- There are two general approaches to explaining the anelastic phenomena described above:

1) Conventional and very broadly used: **The heuristic viscoelastic model.**
   - The key idea is that the time-dependent creep seen in transient experiments and frequency-dependent and phase-delayed responses in harmonic-oscillation experiments are inherent in the material
   - This approach easily explains the above experiments but has problems with physical meanings of material properties and nonphysical wave solutions
   - Also has difficulties with porous rock

This will be described in Lecture 2

2) Less known, maybe more difficult, but correct and much more powerful approach: **Classical continuum mechanics**
   - It has multiple forms. We will use the Lagrangian form
   - It is based on time- and frequency-independent material properties which are not so obvious from observations
   - All interactions are local and instantaneous but depend on gradients and time rates of deformation

We will study this later in the course